Notation and Constants

The symbols and numbers shared by all the orbital mechanics notes

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All the notes use the symbols below. A note that needs a symbol for something else says so where it does, and keeps the meaning local. The last column gives the symbol that Curtis's *Orbital Mechanics for Engineering Students* (the course textbook) uses, where it differs; a dash means no counterpart is listed. Each note's opening section says which other notes it builds on; the study plan gives the order.

Orbits and their integrals

Symbol	Meaning	First defined in	Curtis
$\mu = Gm$	gravitational parameter of the attracting body; $\mu_\odot$, $\mu_\oplus$ for the Sun and the Earth, μ_{Moon}, μ_{Mars} and so on spelled out	N-body	μ

Symbol	Meaning	First defined in	Curtis
$\mathbf{r}$, $\mathbf{v}$; r, v	position and velocity relative to the attracting body, and their lengths	Two-body	same
$\mathbf{h} = \mathbf{r} \times \mathbf{v}$, h	specific angular momentum	Two-body	same
$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$	specific energy	Two-body	same
$\mathbf{e}$, e	eccentricity vector and eccentricity	Two-body	same
$p = h^2/\mu$	semi-latus rectum	Two-body	h^2/μ
$a = -\mu/(2\varepsilon)$	semi-major axis, negative on a hyperbola	Two-body	same
f	true anomaly	Two-body	θ
γ	flight-path angle, from the local horizontal	Two-body	same
v_r, $v_\perp$	radial and transverse parts of $\mathbf{v}$	Two-body	same
E; $M = E - e\sin E$; $n = \sqrt{\mu/a^3}$	eccentric anomaly, mean anomaly, mean motion	Two-body	E; M_e; n
$D = \tan\frac{f}{2}$; $M_p = D + \frac{1}{3}D^3$	parabolic anomaly and its mean anomaly	Two-body	$M_p = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}$, half of ours
H; $M_h = e\sinh H - H$	hyperbolic anomaly and its mean anomaly	Two-body	F; M_h
v_∞; $C_3 = v_\infty^2$	hyperbolic excess speed; characteristic energy	Two-body	same
$f_\infty = \arccos(-1/e)$; $\delta = 2\arcsin(1/e)$	asymptote angle; turning angle	Two-body	θ_∞; δ

Symbol	Meaning	First defined in	Curtis
$b = \lvert a \rvert \sqrt{e^2 - 1}$	aiming radius (impact parameter), written Δ in the two-body note	Two-body	Δ
t_p	time of periapsis passage	Two-body	same

Frames, angles and matrices

Symbol	Meaning	First defined in	Curtis
$\hat{\mathbf{I}}, \hat{\mathbf{J}}, \hat{\mathbf{K}}$	inertial (geocentric equatorial) axes; $\hat{\mathbf{K}}$ along the pole	Orbital elements	same
$\hat{\mathbf{p}}, \hat{\mathbf{q}}, \hat{\mathbf{w}}$	perifocal axes: towards periapsis, 90° ahead, along $\mathbf{h}$	Perifocal	same
i, Ω, ω	inclination, right ascension (or longitude) of the ascending node, argument of periapsis	Perifocal	same
$u = \omega + f$	argument of latitude	Perifocal	same
ϖ, λ	longitude of periapsis, true longitude (for equatorial orbits)	Orbital elements	same
$\mathbf{n} = \hat{\mathbf{K}} \times \mathbf{h}$	node vector, along the ascending node	Orbital elements	$\mathbf{N}$
$\mathbf{R}_1(\theta), \mathbf{R}_3(\theta)$	*active* rotations by θ about the first and third axes	Perifocal	Curtis uses frame rotations, the transposes

Symbol	Meaning	First defined in	Curtis
$\mathbf{Q} = \mathbf{R}_3(\Omega)\mathbf{R}_1(i)\mathbf{R}_3(\omega)$	perifocal components to inertial ones	Perifocal	Curtis tabulates the inverse map, geocentric to perifocal, which is $\mathbf{Q}^\mathsf{T}$
θ_G	Greenwich sidereal angle	Time	—
φ, λ	geodetic latitude and east longitude of a station or a point on the Earth (φ_{gc} is the geocentric latitude)	Ground tracks, Topocentric	—
$\hat{\mathbf{S}}, \hat{\mathbf{E}}, \hat{\mathbf{Z}}$	topocentric axes: south, east, zenith	Topocentric	—
ρ, A, ε	range, azimuth (from north, clockwise) and elevation of a satellite seen from a station	Topocentric	ρ, A, a (elevation)
α, δ	(topocentric) right ascension and declination of a direction	Topocentric	same

The orbit-following frame has three names in the notes, and is one frame. Its axes point radially outwards, along the transverse direction (along the velocity only on a circular orbit) and along $\mathbf{h}$:

Note	Name	Unit vectors	Coordinates
Two-body, perifocal	polar basis	$\hat{\mathbf{r}}, \hat{\theta}, \hat{\mathbf{h}}$	r, f
CW equations, chase	LVLH (local vertical, local horizontal)	$\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$	x, y, z

Note	Name	Unit vectors	Coordinates
Gauss equations	RSW	$\hat{\mathbf{R}}, \hat{\mathbf{S}}, \hat{\mathbf{W}}$	force components a_R, a_S, a_W

Other books order these axes differently (NASA's LVLH, for example, puts x along the velocity and z towards the Earth), which permutes the rows of every matrix written in them.

The five frames and the matrices between them

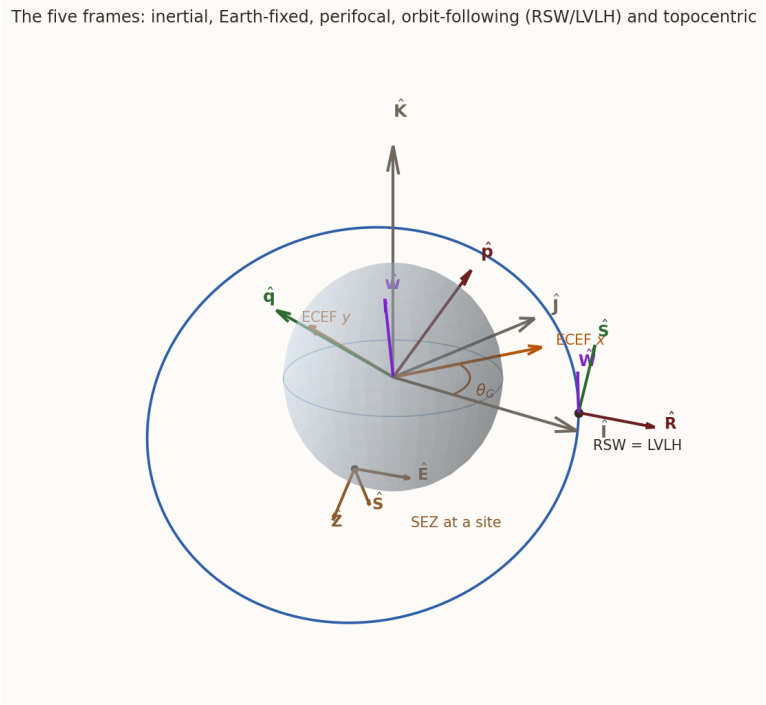


Figure 1: The five frames of the notes. At the Earth's centre: the inertial axes $\hat{\mathbf{I}}, \hat{\mathbf{J}}, \hat{\mathbf{K}}$, the Earth-fixed (ECEF) axes turned by the Greenwich angle θ_G, and the perifocal axes $\hat{\mathbf{p}}, \hat{\mathbf{q}}, \hat{\mathbf{w}}$ of the orbit. On the satellite: the orbit-following frame $\hat{\mathbf{R}}, \hat{\mathbf{S}}, \hat{\mathbf{W}}$ (LVLH, polar basis). At a ground site: the topocentric frame $\hat{\mathbf{S}}, \hat{\mathbf{E}}, \hat{\mathbf{Z}}$ (south, east, zenith). *Script: fig-frames.py*.

Each matrix below takes the components of a vector in the first frame to its components in the second (all rotations active, as in the perifocal note).

From	To	Matrix	Where
perifocal	inertial	$\mathbf{Q} = \mathbf{R}_3(\Omega)\mathbf{R}_1(i)\mathbf{R}_3(\omega)$	Perifocal
orbit-following (RSW)	inertial	$\mathbf{R}_3(\Omega)\mathbf{R}_1(i)\mathbf{R}_3(u)$, with $u = \omega + f$	Gauss equations
inertial	Earth-fixed	$\mathbf{R}_3(-\theta_G)$	Ground tracks, Time for θ_G
Earth-fixed	topocentric (SEZ), at geodetic latitude φ and east longitude λ	rows $\hat{\mathbf{S}}, \hat{\mathbf{E}}, \hat{\mathbf{Z}}$ written in ECEF components: $(\sin\varphi\cos\lambda, \sin\varphi\sin\lambda, -\cos\varphi)$, $(-\sin\lambda, \cos\lambda, 0)$, $(\cos\varphi\cos\lambda, \cos\varphi\sin\lambda, \sin\varphi)$	Topocentric

The reverse maps are the transposes. The angles and ranges measured in the topocentric frame are the input of orbit determination; its symbols are listed below.

Transfers, targeting and perturbations

Symbol	Meaning	First defined in	Curtis
Δv; $v_e = I_{sp}g_0$	velocity change; exhaust speed	Manoeuvres	same
r_{SOI}	radius of a sphere of influence	Patched conics	same
$\Delta\theta$	transfer angle between two position vectors	Lambert	same
$F, G, \dot{F}, \dot{G}$	Lagrange coefficients: $\mathbf{r} = F\mathbf{r}_0 + G\mathbf{v}_0$, $\mathbf{v} = \dot{F}\mathbf{r}_0 + \dot{G}\mathbf{v}_0$	Perifocal	$f, g, \dot{f}, \dot{g}$
χ; $z = \alpha\chi^2$; $C(z)$, $S(z)$	universal anomaly, its square in units of the orbit, Stumpff functions ($\alpha = 1/a$)	Universal variables	same

Symbol	Meaning	First defined in	Curtis
n (in the CW note)	mean motion of the target's circular orbit	CW equations	same
ρ	position of the chaser relative to the target	CW equations	—
J_2; $R_\oplus$	Earth's oblateness coefficient; its equatorial radius	Earth's gravity	J_2; R
$\mathbf{a}_d$	perturbing (disturbing) acceleration	Gauss equations	$\mathbf{p}$
$\mu = m_2/(m_1+m_2)$; C; Ω	mass parameter, Jacobi constant and pseudo-potential of the restricted three-body problem (local to that note)	Restricted three-body	—
$L_1, \dots, L_5$	Lagrange (libration) points	Restricted three-body	same

Observations and orbit determination

The orbit-determination note takes its station, azimuth, elevation, right ascension and declination from the topocentric note (rows above) and adds these.

Symbol	Meaning	First defined in	Curtis
t_1, t_2, t_3; $\tau_1 = t_1 - t_2$, $\tau_3 = t_3 - t_2$, $\tau = t_3 - t_1$	the three observation epochs and the intervals between them; $\tau_1 < 0 < \tau_3$	Orbit determination	—
$\mathbf{R}_i$, $\dot{\mathbf{R}}_i$, $\ddot{\mathbf{R}}_i$	inertial position, velocity and acceleration of the *station* at t_i; $\mathbf{R}_i = \mathbf{R}_3(\theta_G(t_i))\,\mathbf{r}_{\text{site}}$	Orbit determination	—

Symbol	Meaning	First defined in	Curtis
ρ_i, $\mathbf{L}_i$	range and unit line of sight (the $\hat{\rho}$ of the topocentric note), $\mathbf{r}_i = \mathbf{R}_i + \rho_i \mathbf{L}_i$, with $\mathbf{L}_i$ built from α_i, δ_i	Orbit determination, Topocentric	—
$\mathbf{D}$, $\mathbf{N}$, $\mathbf{S}$	Gibbs's vectors: $\mathbf{D} = \mathbf{r}_1 \times \mathbf{r}_2 + \mathbf{r}_2 \times \mathbf{r}_3 + \mathbf{r}_3 \times \mathbf{r}_1$ and $\mathbf{N} = r_1\,\mathbf{r}_2 \times \mathbf{r}_3 + r_2\,\mathbf{r}_3 \times \mathbf{r}_1 + r_3\,\mathbf{r}_1 \times \mathbf{r}_2$ along $\mathbf{h}$, with $p = N/D$; $\mathbf{S} = \mathbf{r}_1(r_2 - r_3) + \mathbf{r}_2(r_3 - r_1) + \mathbf{r}_3(r_1 - r_2)$, perpendicular to $\mathbf{e}$, with $\mathbf{e} = \mathbf{S} \times \mathbf{D}/D^2$	Orbit determination	—
$D_0 = \mathbf{L}_1 \cdot (\mathbf{L}_2 \times \mathbf{L}_3)$	triple product of the three lines of sight; the determinant of Gauss's linear system for the ranges, and a measure of how well the pass is placed	Orbit determination	—
c_1, c_3	weights in $\mathbf{r}_2 = c_1\mathbf{r}_1 + c_3\mathbf{r}_3$ (Gauss), built from the Lagrange coefficients F_i, G_i of the interval from t_2 to t_i	Orbit determination	—
$\mathbf{x}$; H; $\mathsf{P} = \sigma^2(\mathsf{H}^\mathsf{T}\mathsf{H})^{-1}$	least squares: the state at the reference epoch; the matrix $\partial\mathbf{h}/\partial\mathbf{x}$ of the predicted angles; the covariance of $\mathbf{x}$, with σ the standard deviation of one angle	Orbit determination	—

Some of these letters have another meaning elsewhere on this page; the orbit-determination note uses the meaning given here. $\mathbf{R}_i$ with an *index* is the position of the station, while $\mathbf{R}_1(\theta)$, $\mathbf{R}_3(\theta)$ with an *argument* are rotations, and $\hat{\mathbf{R}}$ is the radial axis. $\mathbf{S}$ (bold, no hat) is Gibbs's vector, not the south axis $\hat{\mathbf{S}}$. D, D_0, D_1 and D_{ij} are determinants and triple products, not the parabolic anomaly $D = \tan\frac{f}{2}$. The upright H is a matrix, not the hyperbolic anomaly H, and P is not p. And α and δ are right ascension and declination, never $1/a$ or the turning angle.

Constants

The notes use these values. Some quote $\mu_\oplus$ rounded to 398 600 km^3/s^2, as Curtis does; the difference is below the accuracy of any number they quote.

Quantity	Value
$\mu_\oplus$	398 600.4418 km^3/s^2
$R_\oplus$ (equatorial, WGS-84)	6378.137 km
J_2 (Earth)	1.08263×10^{-3}
$\omega_\oplus$ (rotation against the stars)	7.2921151×10^{-5} rad/s
sidereal day	86 164.0905 s = 23.9345 h
$\mu_\odot$	$1.327\,124\,400\,18 \times 10^{11}$ km^3/s^2
$R_\odot$	6.957×10^5 km
astronomical unit	$1.495\,978\,707 \times 10^8$ km
μ_{Moon}; R_{Moon}; Earth–Moon distance	4902.8 km^3/s^2; 1737.4 km; 384 400 km
μ_{Mars}; R_{Mars}; a_{Mars}	42 828.37 km^3/s^2; 3396.2 km; 1.52368 AU
μ_J; R_J; a_J	$1.266\,865\,34 \times 10^8$ km^3/s^2; 71 492 km; 5.2026 AU
g_0 (for I_{sp})	9.80665 m/s^2