

Indian Missions — Trajectory Problems

How Chandrayaan, Mangalyaan and Aditya-L1 reached their targets, and what the other routes would have cost

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How to use these problems

Every Indian deep-space mission so far has been launched by a rocket that could not send it directly to its target. Chandrayaan-1, Mangalyaan and Aditya-L1 flew on the PSLV, and even the LVM3 (GSLV Mk III) that launched Chandrayaan-2 and -3 placed them in a high elliptical Earth orbit, not on a trajectory to the Moon. Each spacecraft then had to finish the job with its own small engine: a 440 N liquid engine, burning for minutes at a time. The trajectories that ISRO flew are shaped by that constraint, and by the need to save propellant at every step.

Each problem follows the same pattern:

1. **Background:** what the mission was, and the numbers ISRO published.
2. **The trajectory:** a figure, reconstructed from those numbers.
3. **Analysis:** the Δv and the time of each phase, from the published orbits.
4. **Alternatives:** what a direct route, or some other route, would have cost.

Attempt each part before opening its answer. Problem 6 uses the circular restricted three-body problem (CR3BP), and is meant for after it has been covered in class; the others need only the two-body problem, patched conics and the rocket equation.

#	mission	year	what it shows
1	Chandrayaan-1	2008	reaching the Moon on a PSLV: many small perigee burns
2	Chandrayaan-2	2019	what a bigger launcher buys; capturing safely at the Moon

#	mission	year	what it shows
3	Chandrayaan-3	2023	a staged lunar capture that costs nothing extra; bringing the module home
4	Mangalyaan (MOM)	2013	a failed burn that cost no propellant; the launch window; the Mars orbit
5	Aditya-L1 (two-body)	2023	escaping to 1.5 million km, and why the cruise took 110 days
6	Aditya-L1 (CR3BP)	2023–24	the halo orbit, and why a slow transfer makes the insertion almost free

Data. $\mu_{\oplus} = 398\,600.4418 \text{ km}^3/\text{s}^2$, $R_{\oplus} = 6378.137 \text{ km}$; Moon: $\mu_{\text{M}} = 4902.80 \text{ km}^3/\text{s}^2$, $R_{\text{M}} = 1737.4 \text{ km}$, mean distance $r_{\text{M}} = 384\,400 \text{ km}$; Mars: $\mu = 42\,828.37 \text{ km}^3/\text{s}^2$, $R = 3396.2 \text{ km}$; Sun: $\mu_{\odot} = 1.32712 \times 10^{11} \text{ km}^3/\text{s}^2$, $1 \text{ AU} = 1.495\,979 \times 10^8 \text{ km}$; $g_0 = 9.80665 \text{ m/s}^2$.

Engine. All five spacecraft used a 440 N liquid apogee motor. Take $I_{sp} = 310 \text{ s}$, so $v_e = I_{sp}g_0 = 3.040 \text{ km/s}$ and the propellant flow is $\dot{m} = F/v_e = 0.1447 \text{ kg/s}$.

Assumptions (unless a part says otherwise). Orbits are given as *perigee* $\times$ *apogee altitude*. Each apogee-raising burn is impulsive and tangential, at the perigee of the orbit before it. Successive Earth-bound orbits share one apse line. The arrival at the Moon is a coplanar patched conic: the transfer orbit’s apogee is at the Moon’s distance, and the Moon moves on a circle.

All the numbers in the answers are recomputed by [codes/indian-missions-verify.py](#), using the models in [codes/_imcommon.py](#). The figures are reconstructions from the published orbits, not flight data.

Problems

1. Chandrayaan-1: reaching the Moon on a PSLV

Background. Chandrayaan-1, India’s first mission to the Moon, was launched on 22 October 2008 by PSLV-C11, the first flight of the more powerful PSLV-XL. The PSLV could lift the 1380 kg spacecraft only into an elliptical orbit of $255 \times 22\,860 \text{ km}$, a little smaller than a geostationary transfer orbit. Over the next thirteen days the spacecraft’s own 440 N engine raised the apogee in five burns, each made near perigee, the last of which (4 November) put it on the lunar transfer trajectory. On 8 November an 817 s burn captured it into a $504 \times 7502 \text{ km}$ lunar orbit, and four more burns brought it down to a 100 km circular polar orbit by 12 November. Its mass in lunar orbit was 675 kg.

date (2008)	event	orbit after [km]	reported burn
22 Oct	PSLV injection	$255 \times 22\,860$	—
23 Oct	Earth-bound burn 1	$305 \times 37\,900$	about 18 min
25 Oct	Earth-bound burn 2	$336 \times 74\,715$	about 16 min
26 Oct	Earth-bound burn 3	$348 \times 164\,600$	about 9.5 min
29 Oct	Earth-bound burn 4	$465 \times 267\,000$	about 3 min
4 Nov	lunar transfer burn	apogee about 380 000 (take perigee 465)	about 2.5 min

date (2008)	event	orbit after [km]	reported burn
8 Nov	lunar orbit insertion	504×7502 (Moon)	817 s
9–12 Nov	four lowering burns	200×7502 , 187×255 , 101×255 , 100×100	—

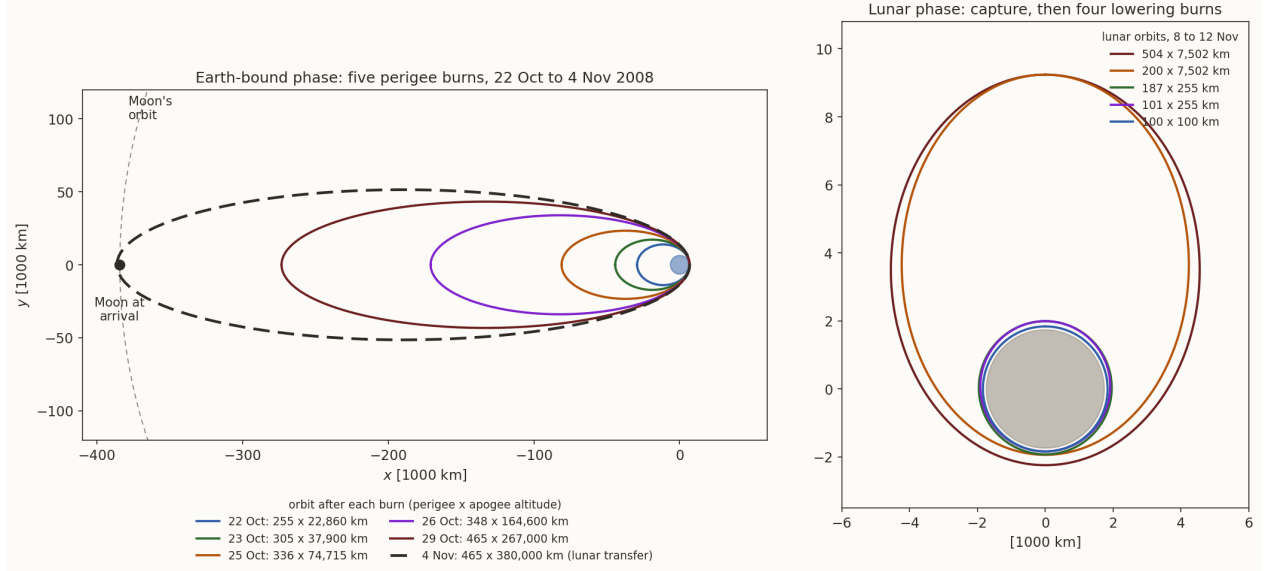


Figure 1: Chandrayaan-1's orbits, reconstructed from the published figures. Left: the Earth-bound phase, to scale. Right: the lunar orbits, to scale. [Script](#).

(a) Find the impulsive Δv of each of the five Earth-bound burns, and their total. Show that, to within a few m/s, the total is what a single burn at perigee from the launch orbit to the lunar transfer orbit would need. Why must that be so?

Answer. Each burn is tangential at perigee, so it changes only the speed at that point. Before the burn the speed is that of the old orbit at its perigee; after it, that of the new orbit at the same radius. *Vis-viva* gives both. For burn 1, at $r_b = R_\oplus + 255 = 6633.1$ km:

$$a_{\text{old}} = \frac{1}{2}(2R_\oplus + 255 + 22\,860) = 17\,935.6 \text{ km}, \quad a_{\text{new}} = \frac{1}{2}(2R_\oplus + 305 + 37\,900) = 25\,480.6 \text{ km},$$

$$\Delta v_1 = \sqrt{\mu \left(\frac{2}{r_b} - \frac{1}{a_{\text{new}}} \right)} - \sqrt{\mu \left(\frac{2}{r_b} - \frac{1}{a_{\text{old}}} \right)} = 10.225 - 9.898 = 327.0 \text{ m/s}.$$

The same steps for the other four burns give

burn	orbit before $\rightarrow$ after	Δv [m/s]
1	$255 \times 22\,860 \rightarrow 305 \times 37\,900$	327.0
2	$305 \times 37\,900 \rightarrow 336 \times 74\,715$	317.4
3	$336 \times 74\,715 \rightarrow 348 \times 164\,600$	217.1
4	$348 \times 164\,600 \rightarrow 465 \times 267\,000$	76.6
5	$465 \times 267\,000 \rightarrow 465 \times 380\,000$	38.3
total		976.4

A single burn at 255 km from the launch orbit to a $255 \times 380\,000$ km orbit needs 972.5 m/s. The two agree because every burn is made at (nearly) the same point and along the velocity: the speed there just steps up, $v_0 \rightarrow v_1 \rightarrow \dots \rightarrow v_5$, and the sum of the steps is $v_5 - v_0$ whatever the number of steps. The 4 m/s difference comes only from the perigee creeping up from 255 to 465 km.

(b) Find the period of each Earth-bound orbit. Using the dates in the table, how many times did the spacecraft pass perigee in each orbit before the next burn? How long would the lunar transfer take on a Kepler ellipse, and how does that compare with the four days from 4 to 8 November?

Answer. $T = 2\pi\sqrt{a^3/\mu}$:

orbit	a [km]	T [h]	days to next burn	revolutions
$255 \times 22\,860$	17 936	6.64	1	≈ 3.6
$305 \times 37\,900$	25 481	11.24	2	≈ 4.3
$336 \times 74\,715$	43 904	25.43	1	≈ 1
$348 \times 164\,600$	88 852	73.2	3	≈ 1
$465 \times 267\,000$	140 111	145.0	6	≈ 1

In the small orbits there are several perigee passes to choose from. In the large ones there is only one per period, and the burn has to wait for it: the Earth-bound phase is slow because the orbits become slow.

The lunar transfer orbit ($465 \times 380\,000$ km, $a = 196\,600$ km) has a half-period of $\pi\sqrt{a^3/\mu} = 5.0$ days. The actual trip took about four days because, on its last part, the spacecraft fell towards the Moon, whose pull our two-body ellipse leaves out.

(c) **Alternative: one long burn.** The engine is small, so each burn lasts minutes. Using the rocket equation with $F = 440$ N and $I_{sp} = 310$ s, estimate the duration of each of the five burns, and compare with the reported durations. Then consider making the whole 976 m/s in one burn, centred on perigee, from the launch orbit. A numerical integration of that burn (with the thrust along the velocity) gives a burn of 2931 s that costs 1116 m/s and lifts the perigee to 683 km. Explain why it costs more.

Answer. For a burn of Δv starting at mass m_0 , the mass drops to $m_0 e^{-\Delta v/v_e}$ at the rate $\dot{m} = F/v_e$, so

$$t_b = \frac{m_0 v_e}{F} (1 - e^{-\Delta v/v_e}).$$

Burn 1 starts at $m_0 = 1380$ kg: $m_0 v_e / F = 9535$ s and $1 - e^{-0.327/3.040} = 0.1020$, so $t_b \approx 973$ s. Carrying the mass down the chain, and integrating each burn numerically (which also accounts for the finite arc), gives

burn	Δv (impulsive)	integrated burn	reported
1	327 m/s	993 s	about 18 min (1080 s)
2	317 m/s	860 s	about 16 min (960 s)
3	217 m/s	531 s	about 9.5 min (570 s)
4	77 m/s	177 s	about 3 min
5	38 m/s	86 s	about 2.5 min

The agreement is good: the small engine and the published orbits tell the same story. The five integrated burns cost 988.8 m/s in all, only 12 m/s more than the impulsive total.

A single burn of 2931 s (49 min) is only 12% of the launch orbit's 6.6 h period, but it is centred on perigee, where the spacecraft moves fastest: by Kepler's equation it spans 176° of true anomaly, from 88° before perigee to 88° after. The spacecraft sweeps halfway round its orbit while the engine fires, so much of the thrust is applied well away from perigee, where the speed is lower and each m/s buys less energy ($\Delta\varepsilon = v \Delta v$). Part of it also goes into raising the perigee instead of the apogee. The result is 1116 m/s, 13% more. Splitting the burn into five short ones, each centred on a perigee, keeps every burn close to the point where it is most effective. The price is time: thirteen days instead of one orbit.

(d) **Alternative: a direct trans-lunar injection.** Suppose instead that the launcher had left Chandrayaan-1 in a 255 km circular orbit, and the spacecraft had to make the whole trans-lunar injection itself. What Δv would that need, and what fraction of the spacecraft's mass would be propellant? How much of this did the PSLV supply by placing it in $255 \times 22\,860$ km?

Answer. At $r = 6633.1$ km the circular speed is $\sqrt{\mu/r} = 7.752$ km/s, and the speed on a $255 \times 380\,000$ km orbit is 10.870 km/s. So a direct injection needs $\Delta v = 3118$ m/s, i.e. a propellant fraction $1 - e^{-3.118/3.040} = 64\%$: about 890 kg of the 1380 kg, before any of the lunar burns. That was not possible.

The PSLV's elliptical orbit already has a perigee speed of 9.898 km/s, so the launcher supplied $9.898 - 7.752 = 2146$ m/s of the 3118 m/s; the spacecraft added the remaining 972 m/s at perigee. This is the general strategy of all five missions in this set: let the launcher do as much as it can, at perigee, and finish at perigee with the spacecraft's engine.

(e) Capture at the Moon. Find the arrival excess speed v_∞ at the Moon for a transfer with perigee 465 km. Then find (i) the insertion burn into 504×7502 km, (ii) the total of the insertion and the four lowering burns, and (iii) the single burn that would put the spacecraft directly into the 100 km circular orbit. Why was the staged route worth its extra cost? Compare the impulsive insertion with what an 817 s burn delivers.

Answer. Arrival. On the transfer orbit ($r_p = 6843.1$ km, apogee at r_M), the speed at the Moon's distance is $\sqrt{\mu(2/r_M - 1/a)} = 0.190$ km/s. The Moon moves at $\sqrt{\mu/r_M} = 1.018$ km/s in the same direction, so relative to the Moon the spacecraft arrives at $v_\infty = 0.828$ km/s.

At perilune. On the approach hyperbola the speed at radius r is $\sqrt{v_\infty^2 + 2\mu_M/r}$. Each capture burn is that minus the speed on the target orbit at the same r .

- (i) At $r = R_M + 504 = 2241.4$ km: hyperbola 2.249 km/s, orbit 504×7502 km 1.876 km/s, so $\Delta v_{\text{LOI}} = 373$ m/s.
- (ii) The four lowering burns, each tangential at the apsis that the two orbits share: 26 m/s (perilune lowered to 200 km at the 7502 km apolune), 446 m/s (apolune lowered to 255 km at the 200 km perilune), 18 m/s and 33 m/s. Total with insertion: 897 m/s.
- (iii) Directly into the 100 km circle: hyperbola 2.454 km/s at $r = 1837.4$ km, circular speed 1.634 km/s, so 820 m/s.

The staged route costs about 76 m/s more, because the insertion was made at 504 km, not at 100 km, where the spacecraft is faster and each m/s is worth more. It buys safety. The smallest burn that captures at all is $v_{\text{hyp}} - v_{\text{esc}} = 158$ m/s at 504 km, so the 373 m/s insertion would still have captured the spacecraft even if the engine had delivered less than half of it. The burn is also shorter, so its finite-burn losses are smaller, and the high orbit gives the ground days to measure it before the next burn.

Check. After the integrated Earth-bound burns the mass is 997 kg. An 817 s burn uses $0.1447 \times 817 = 118$ kg, so it delivers $3.040 \ln(997/879) = 384$ m/s: the impulsive 373 m/s plus about 3% of finite-burn loss.

(f) The mass budget. From 1380 kg at launch to 675 kg in lunar orbit, what Δv does the rocket equation give? Compare it with your impulsive total for the Earth-bound and lunar phases, and list what could account for the difference.

Answer. $\Delta v = v_e \ln(1380/675) = 3.040 \times 0.715 = 2174$ m/s. The impulsive estimates give $976 + 897 = 1873$ m/s, which leaves about 300 m/s (14%) unaccounted for. Candidates:

- finite-burn losses (12 m/s on the Earth-bound burns, about 11 m/s on the insertion);
- trajectory-correction manoeuvres on the way to the Moon;
- propellant used by the 22 N attitude-control thrusters, whose I_{sp} is lower than the main engine's;
- the burns were not exactly tangential, nor exactly at the apsides;
- the assumed $I_{sp} = 310$ s, and the exact meaning of "mass in lunar orbit".

Even so, the published orbits explain about 86% of the propellant that was used.

2. Chandrayaan-2: what a bigger launcher buys

Background. Chandrayaan-2 (orbiter, the Vikram lander and the Pragyan rover; 3850 kg in all) was launched on 22 July 2019 by the first operational GSLV Mk III (LVM3-M1), after a launch attempt on 15 July was called off less than an hour before lift-off. The launcher placed it in a $169.7 \times 45\,475$ km orbit. The first Earth-bound burn (48 s, 24 July) mainly trimmed the perigee;

four more raised the apogee, and the trans-lunar injection of 14 August (1203 s) sent it to the Moon. On 20 August a 1738 s burn captured it into a $114 \times 18\,072$ km lunar orbit, and four lowering burns brought it to 119×127 km by 1 September. (Vikram was lost during its landing attempt on 7 September; the orbiter carried on operating.)

date (2019)	event	orbit after [km]
22 Jul	GSLV Mk III injection	$169.7 \times 45\,475$
24 Jul	burn 1 (48 s, trim)	$230 \times 45\,163$
26 Jul – 6 Aug	burns 2–5	$251 \times 54\,829$, $276 \times 71\,792$, $277 \times 89\,472$, $276 \times 142\,975$
14 Aug	trans-lunar injection (1203 s)	target $266 \times 413\,623$
20 Aug	lunar orbit insertion (1738 s)	$114 \times 18\,072$ (Moon)
21 Aug – 1 Sep	four lowering burns	118×4412 , 179×1412 , 124×164 , 119×127

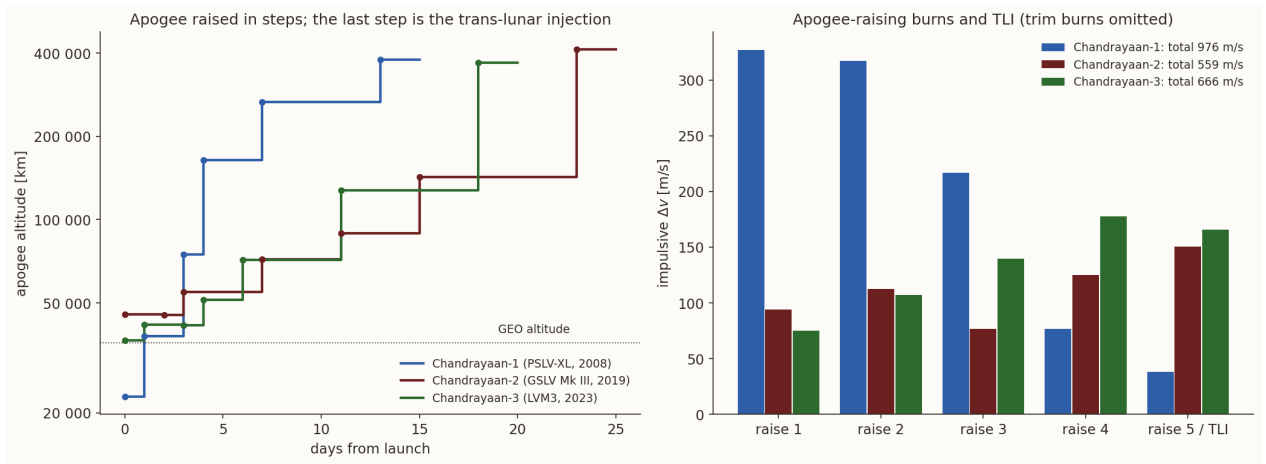


Figure 2: The Earth-bound phases of the three Chandrayaans. Left: apogee against days from launch. Right: the impulsive Δv of each apogee-raising burn and of the trans-lunar injection, from the published orbits (the trim burns that only moved the perigee are left out). [Script](#).

(a) Find the impulsive Δv of Chandrayaan-2’s apogee-raising burns and its trans-lunar injection (leave out the 48 s trim burn), and compare the total with Chandrayaan-1’s and Chandrayaan-3’s (Figure 2). How much propellant did each spacecraft spend in its Earth-bound phase? (Chandrayaan-3: 3900 kg at launch.)

Answer. The method of Problem 1(a), burn by burn at the perigee of the orbit before it, gives 94.2, 112.4, 76.9, 125.3 m/s for the raising burns and 150.3 m/s for the injection: 559 m/s in all. With the rocket equation, $m_{\text{prop}} = m_0(1 - e^{-\Delta v/v_e})$:

mission	launch orbit [km]	Earth-bound Δv	m_0	propellant
Chandrayaan-1	$255 \times 22\,860$	976 m/s	1380 kg	379 kg
Chandrayaan-2	$169.7 \times 45\,475$	559 m/s	3850 kg	647 kg
Chandrayaan-3	$170 \times 36\,500$	666 m/s	3900 kg	768 kg

The spacecraft’s share is set by how much energy the launcher left it with. The energy of the launch orbit, $\varepsilon = -\mu/2a$, is -11.1 , -6.8 and -8.1 km^2/s^2 for the three missions: the GSLV Mk III’s orbit for Chandrayaan-2 was the most energetic, and the spacecraft’s own Δv was the smallest. A higher apogee from the launcher saves Chandrayaan-2 about 107 m/s against Chandrayaan-3, which on a 3850 kg spacecraft is about 130 kg of propellant.

(b) How much Δv did each launcher give beyond a circular orbit at its injection perigee? Why do the launchers stop short of a trans-lunar injection, when their upper stages could in principle burn longer?

Answer. At the injection perigee r_p , the launcher's contribution beyond circular speed is $v_p - \sqrt{\mu/r_p}$:

- Chandrayaan-1 (PSLV-XL): 2146 m/s;
- Chandrayaan-2 (GSLV Mk III): 2595 m/s;
- Chandrayaan-3 (LVM3): 2475 m/s.

A trans-lunar injection from about 170 km needs roughly 3.1 km/s beyond circular speed, so the launcher would have to supply another 0.6 to 1 km/s. The upper stage is a heavy, inefficient mass to accelerate that far; every m/s it adds is paid for by a smaller payload. The spacecraft's engine is lighter, and it can make its share in several burns, each at perigee, with tracking in between. The cost is time: compare the 23 days of Chandrayaan-2's Earth-bound phase with the three days of an Apollo trans-lunar coast, whose Saturn V upper stage did the whole injection.

(c) **Capture with a safety margin.** Find Chandrayaan-2's arrival v_∞ (transfer perigee 276 km), its insertion Δv into 114×18072 km, and the smallest burn that would have captured it at all. What fraction of the insertion burn could the engine have lost without the spacecraft escaping? Check the insertion against the 1738 s burn.

Answer. As in Problem 1(e): $v_\infty = 0.830$ km/s. At $r = R_M + 114 = 1851.4$ km the hyperbola speed is $\sqrt{0.830^2 + 2(4902.8)/1851.4} = 2.447$ km/s and the orbit speed is 2.201 km/s, so $\Delta v_{\text{LOI}} = 246$ m/s.

The escape speed at that radius is $\sqrt{2\mu_M/r} = 2.301$ km/s, so any burn above $2.447 - 2.301 = 145$ m/s captures. The engine could have delivered only $145/246 = 59\%$ of the burn and the spacecraft would still have stayed at the Moon, in a very long orbit that later burns could correct. Capturing into a *high* ellipse gives this margin; a direct capture into a low circle leaves none, because then the whole 820 m/s must be delivered correctly in one burn.

Check. The mass at insertion is $3850e^{-0.559/3.040} = 3203$ kg. A 1738 s burn uses 252 kg, so it delivers $3.040 \ln(3203/2952) = 249$ m/s, against the impulsive 246 m/s.

(d) **Alternative: capture directly into the final orbit.** Find the total Δv of the insertion and the four lowering burns (take each lowering burn at the perilune of the orbit before it), and the single burn that would capture directly into 119×127 km. How long would that single burn last?

Answer. The four lowering burns cost 183, 185, 199 and 9 m/s, so the staged total is $246 + 576 = 822$ m/s. A direct capture into 119×127 km needs 817 m/s. The staged route costs only 5 m/s more, because, unlike Chandrayaan-1's, all its burns are at nearly the same low perilune: the speeds at that point step down, and the steps add up to the same total (Problem 1(a) in reverse).

A single 817 m/s burn from 3203 kg would last $\frac{3203(3.040)}{0.440} (1 - e^{-0.817/3.040}) = 5215$ s, i.e. 87 minutes, against an orbit whose perilune passage lasts a few minutes. Its finite-burn losses would be far larger, and it would have no margin at all. Splitting the capture costs almost nothing and removes both problems.

3. Chandrayaan-3: a staged capture, and bringing the module home

Background. Chandrayaan-3 (3900 kg: a 2148 kg propulsion module carrying the Vikram lander and the Pragyan rover) was launched by LVM3-M4 on 14 July 2023 into a 170×36500 km orbit. After the Earth-bound burns and the trans-lunar injection of 1 August, a 1835 s burn on 5 August captured it into a 164×18074 km lunar orbit. Four lowering burns brought it to 153×163 km by 16 August; the lander separated on 17 August, lowered its own orbit to 25×134 km, and landed near the south pole on 23 August.

ISRO later reported that the precise injection by LVM3 and the planning of the Earth and lunar burns had left over 100 kg of propellant in the propulsion module (dry mass 448.6 kg). That was

used to bring the module back to an Earth orbit: on 9 October 2023 its apolune was raised from 150 km to 5112 km (period from 2.1 h to 7.2 h), and on 13 October a trans-Earth injection sent it away from the Moon. After four lunar flybys it left the Moon's sphere of influence on 10 November, and by early December it was in an Earth orbit with a first perigee of about 154 000 km, a period of about 13 days and an inclination of 27° . The planned orbit had been about $1.8 \text{ lakh} \times 3.8 \text{ lakh km}$, chosen to keep clear of the GEO belt and of the Moon.

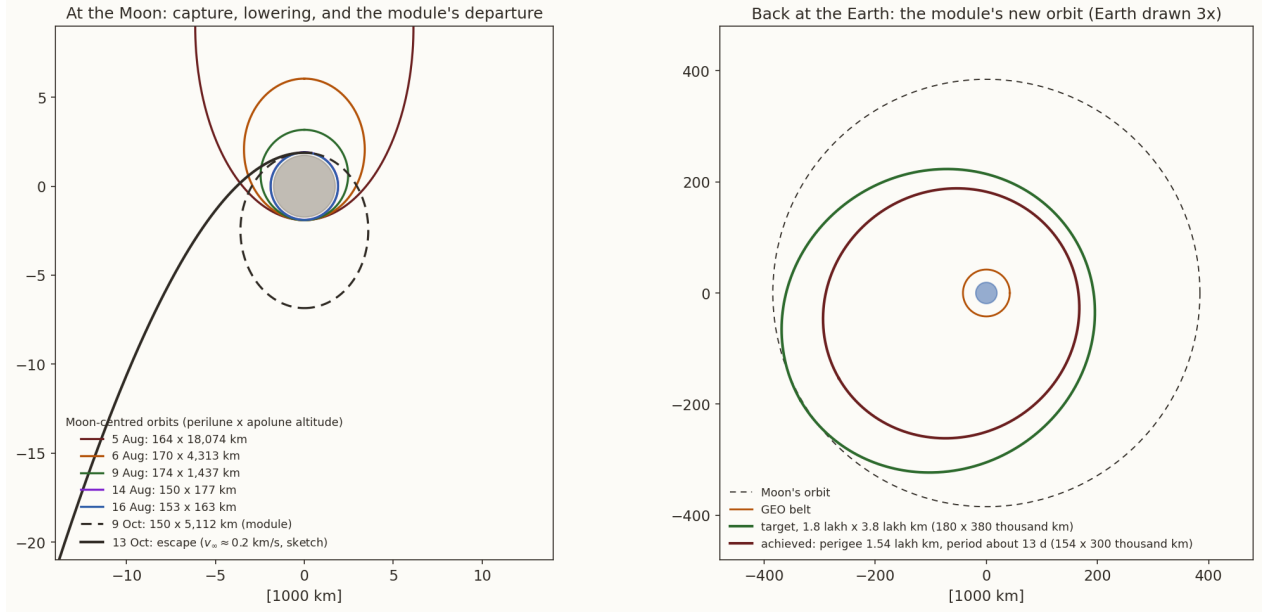


Figure 3: Chandrayaan-3 at the Moon and the propulsion module's return, reconstructed. Left: the lunar orbits to scale, with the module's departure. Right: the Earth orbit it was moved to. [Script](#).

(a) Find the insertion Δv (transfer perigee 288 km) and the four lowering burns, each taken at the perilune of the orbit before it. Compare their total with a single capture into $153 \times 163 \text{ km}$. How long would that single burn have lasted? Compare the insertion with what the 1835 s burn delivers.

Answer. $v_\infty = 0.830 \text{ km/s}$. At $r = R_M + 164 = 1901.4 \text{ km}$ the hyperbola speed is 2.418 km/s and the $164 \times 18\,074 \text{ km}$ orbit's speed is 2.169 km/s , so $\Delta v_{\text{LOI}} = 249 \text{ m/s}$. The lowering burns then cost 188, 185, 192 and 2 m/s, so the staged total is 816 m/s, against 812 m/s for a direct capture into $153 \times 163 \text{ km}$: a penalty of 4 m/s.

The mass at insertion is about $3900e^{-0.666/3.040} = 3132 \text{ kg}$. A single 812 m/s burn would last 5070 s (85 min), against the 1835 s that ISRO's insertion took. The 1835 s burn uses 266 kg and delivers $3.040 \ln(3132/2866) = 269 \text{ m/s}$, about 20 m/s more than the impulsive estimate; that is the order of the finite-burn loss, and of the uncertainty in our coplanar arrival model.

(b) The lander's two deboost burns took it from $153 \times 163 \text{ km}$ to $113 \times 157 \text{ km}$ and then to $25 \times 134 \text{ km}$. Assuming each is a tangential burn at the apolune, find their Δv . Why lower the perilune to 25 km before starting the powered descent?

Answer. At the apolune the burn lowers the perilune and leaves the apolune where it is, so it is placed there. The two burns cost about 10 m/s and 25 m/s; the second is larger because it lowers the perilune by much more. (The two orbits in each pair do not share their apolune exactly, which the model ignores.)

The powered descent must remove the whole orbital speed, about 1.7 km/s , with the lander's own engines. Starting it at the perilune of a $25 \times 134 \text{ km}$ orbit means starting low, where the remaining descent is short and the spacecraft's speed is close to what the descent would have to cancel anyway, so little is lost to fighting gravity; and the deboost itself is cheap, because it is made at the apolune, where the speed is lowest.

(c) Bringing the module home. Check ISRO's periods of 2.1 h and 7.2 h. Then estimate the return with a patched conic: find the module's v_∞ relative to the Moon for an Earth orbit of $180\,000 \times 380\,000$ km whose apogee is at the Moon's distance, and the Δv of the two burns. How much propellant does that need, and does it fit "over 100 kg"? Why make it in two burns, when one burn at the same point would cost the same?

Answer. *Periods.* The 150 km circle has $r = 1887.4$ km: $T = 2\pi\sqrt{r^3/\mu_M} = 2.04$ h. The 150×5112 km orbit has $a = 4368.4$ km: $T = 7.20$ h. Both match.

The target orbit. With $r_p = R_\oplus + 180\,000$ and $r_a = R_\oplus + 380\,000$, the speed at apogee is $\sqrt{2\mu r_p/(r_a(r_a + r_p))} = 0.819$ km/s, against the Moon's 1.018 km/s. The module must leave the Moon at $v_\infty = 0.199$ km/s, backwards relative to the Moon's motion.

The burns. At $r = 1887.4$ km: the circular speed is 1.612 km/s, the 150×5112 km orbit has 2.018 km/s at perilune, and the escape hyperbola needs $\sqrt{0.199^2 + 2(4902.8)/1887.4} = 2.288$ km/s. So the apolune raise costs 406 m/s and the trans-Earth injection 270 m/s: 676 m/s in all. (Merely escaping the Moon would cost $(\sqrt{2} - 1) \times 1.612 = 668$ m/s.)

Propellant. With the dry mass 448.6 kg left at the end, $m_{\text{prop}} = 448.6(e^{0.676/3.040} - 1) = 112$ kg: "over 100 kg" is just enough. The margin is thin, and the four lunar flybys of the real trajectory, which exchange energy with the Moon for free, are part of how ISRO made it work.

Two burns. Both are at the perilune, so impulsively they cost the same as one. But the first puts the module on a 7.2 h orbit, which lets the second be timed so that the module leaves the Moon in the right direction, and halves the duration of each burn.

(d) ISRO reported a first perigee of about 154 000 km altitude and a period of nearly 13 days. What apogee does that imply? Compare with the planned orbit, and with the alternatives: leaving the module in lunar orbit, or crashing it on the Moon.

Answer. $a = (\mu T^2/4\pi^2)^{1/3} = 233\,500$ km for $T = 13$ d, so $r_a = 2a - r_p = 2(233\,500) - 160\,400$, an apogee altitude of about 300 000 km. The planned $180\,000 \times 380\,000$ km orbit would have had a period of 17.7 days. The achieved orbit is smaller, with its apogee inside the Moon's orbit; either way its perigee is far above the GEO belt at 35 786 km, which was the constraint.

The alternatives are cheaper. Lowering the perilune from the 150 km circle to the surface costs only 34 m/s, and leaving the module where it was costs nothing. The return was worth its 676 m/s because it let the module's SHAPE instrument keep observing the Earth from far away, and, in ISRO's words, to gather additional information for future lunar missions: it was a rehearsal of leaving lunar orbit and returning to the Earth.

4. Mangalyaan: Mars on a PSLV

Background. The Mars Orbiter Mission (Mangalyaan, 1337 kg at launch, of which 852 kg was propellant) was launched by PSLV-C25 on 5 November 2013. The GSLV, which could have placed it in a higher orbit, was not yet flying reliably, and the PSLV could only put it into a $264 \times 23\,904$ km orbit. Six perigee burns raised the apogee to 192 874 km. The fourth one fell short: the engine delivered 35 m/s instead of the planned 130 m/s, raising the apogee to 78 276 km instead of about 100 000 km, and a supplementary burn the next day took it to 118 642 km. The trans-Mars injection (TMI) of 1 December 2013 IST lasted 1329 s. After a 298-day cruise, a 24-minute burn on 24 September 2014 put the spacecraft into a $421.7 \times 76\,993.6$ km Mars orbit with a period of 72 h 51 min, and only about 40 kg of propellant was left.

date (2013)	event	apogee after [km]
5 Nov	PSLV-C25 injection	23 904 (perigee 264)
7, 8, 9 Nov	burns 1–3	28 825, 40 186, 71 636
11 Nov	burn 4 (short)	78 276
12 Nov	supplementary burn	118 642

date (2013)	event	apogee after [km]
16 Nov	burn 5	192 874
1 Dec	trans-Mars injection	escape

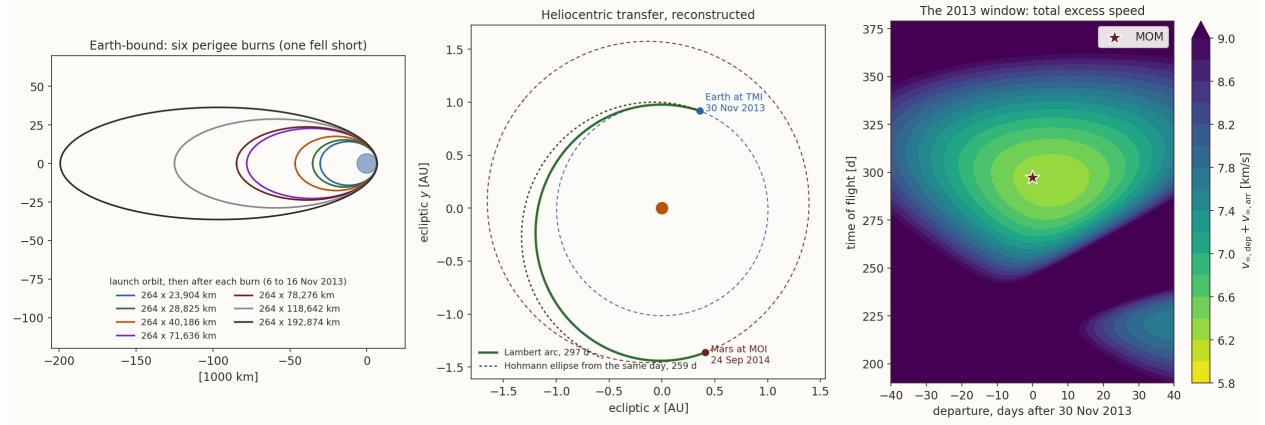


Figure 4: The Mars Orbiter Mission, reconstructed. Left: the Earth-bound orbits, to scale (perigee held at 264 km). Middle: the Lambert arc between the Earth on 30 November 2013 (UTC) and Mars on 24 September 2014, with the Hohmann ellipse from the same day. Right: the departure plus arrival excess speed over the 2013 window. [Script](#).

(a) Find the impulsive Δv of the six Earth-bound burns (hold the perigee at 264.1 km). Then compare the *planned* sequence, 71 636 $\rightarrow$ 100 000 $\rightarrow$ 118 642 km, with the *flown* one, 71 636 $\rightarrow$ 78 276 $\rightarrow$ 118 642 km. How much did the short burn cost?

Answer. The six burns cost 127.2, 200.4, 268.1, 32.5, 126.1 and 101.8 m/s: 856 m/s in all.

The planned pair would have been 111.8 m/s (to 100 000 km) and 46.9 m/s (on to 118 642 km); the flown pair was 32.5 m/s and 126.1 m/s. Both total 158.6 m/s. All four burns are tangential at the same perigee, so the speed there just steps from its value on the 71 636 km orbit to its value on the 118 642 km orbit, however the steps are divided (Problem 1(a)). The short burn cost **no** extra Δv at all, only time: one extra perigee pass, about a day, which the schedule could absorb before the TMI date.

(b) **The heliocentric transfer.** The Lambert arc between the Earth’s position at TMI (30 November 2013, 19:19 UTC) and Mars’s position at arrival (24 September 2014, 02:10 UTC) takes 297.3 days and needs a departure excess speed of 3.129 km/s ($C_3 = 9.79 \text{ km}^2/\text{s}^2$) and an arrival excess speed of 3.155 km/s. Compare with a Hohmann transfer between circular orbits of 1 and 1.524 AU. Why did MOM not fly a Hohmann transfer, and why did it take longer?

Answer. The Hohmann transfer has $a_t = 1.262 \text{ AU}$; *vis-viva* gives departure and arrival excess speeds of 2.945 and 2.649 km/s, and the transfer takes half its period, 259 days.

A Hohmann transfer needs Mars to be exactly 180° round from the Earth’s departure point when the spacecraft arrives, and it assumes circular, coplanar orbits. Mars’s orbit has $e = 0.093$ and is inclined by 1.85° , and in 2013 the planets were not in that ideal geometry. The arc that MOM actually needed swept 218° round the Sun, more than half a revolution (a “type II” transfer), and that longer arc took 297 days. Its excess speeds are 0.18 and 0.51 km/s higher than the Hohmann ideal.

The right panel of Figure 4 shows that MOM’s dates were nevertheless very close to the best available: the lowest total $v_{\infty, \text{dep}} + v_{\infty, \text{arr}}$ in the 2013 window is 6.24 km/s, for a departure five days later with a 295-day flight, against MOM’s 6.28 km/s.

(c) Find the impulsive TMI Δv from the final Earth orbit, using $v_\infty = 3.129 \text{ km/s}$. Compare it with a TMI made directly from the launch orbit, and from a 264 km circular parking orbit. What did the six Earth-bound burns buy?

Answer. At perigee ($r = 6642.2$ km) the escape hyperbola needs $\sqrt{v_\infty^2 + 2\mu/r} = \sqrt{9.792 + 120.02} = 11.394$ km/s.

- From the 264×192874 km orbit ($v_p = 10.777$ km/s): 616 m/s. (The flown burn was reported as 648 m/s; the difference is the finite-burn loss of a 22-minute burn.)
- From the launch orbit ($v_p = 9.921$ km/s): 1472 m/s, in one burn.
- From a 264 km circle (7.747 km/s): 3647 m/s.

The six burns and the TMI together cost $856 + 616 = 1472$ m/s, the same as one burn from the launch orbit (all are at perigee again). What they bought is that no single burn was longer than about 22 minutes, and that the departure could wait in a slowly growing orbit until the right day. A single 1472 m/s burn would have lasted about 45 minutes and lost far more to finite-burn effects (compare Problem 1(c)).

(d) The Mars orbit. Find the Mars-orbit insertion Δv into 421.7×76993.6 km, and check the orbit's period. What would a capture into a 400 km circular orbit have cost? The spacecraft weighed about 815 kg after TMI and had about 330 kg of propellant left. Could it have done that?

Answer. At $r = 3396.2 + 421.7 = 3817.9$ km: the hyperbola speed is $\sqrt{3.155^2 + 2(42828)/3817.9} = 5.691$ km/s and the orbit speed (with $a = 42104$ km) is 4.628 km/s, so $\Delta v_{\text{MOI}} = 1063$ m/s. The period is $2\pi\sqrt{a^3/\mu} = 72.86$ h, which is ISRO's 72 h 51 min.

Into a 400 km circle: $\Delta v = 5.702 - 3.359 = 2343$ m/s.

The propellant for each: $m_{\text{prop}} = 815(1 - e^{-\Delta v/v_e})$. The flown orbit needs 241 kg, which fits in 330 kg (ISRO reported about 40 kg left after MOI, the rest having gone to corrections and finite-burn losses). The circular orbit would need 438 kg: impossible. A very elliptical orbit is the cheapest way to be captured, because only a little speed has to be removed at a periapsis that is reached at high speed. The high apoapsis also let MOM's colour camera take whole-disc images of Mars.

(e) The mass budget. From 1337 kg at launch, with 852 kg of propellant of which about 40 kg was left after MOI, what total Δv did MOM make? Compare it with the sum of your impulsive estimates.

Answer. The mass after MOI was $1337 - 812 = 525$ kg, so $\Delta v = 3.040 \ln(1337/525) = 2841$ m/s. The impulsive estimates add up to $856 + 616 + 1063 = 2535$ m/s; the flown TMI alone was 32 m/s more than impulsive, and the MOI, a 24-minute burn, will have lost a comparable amount. The remaining 250 m/s or so went into trajectory corrections, attitude control, and the departures of the real burns from our idealised tangential, coplanar ones.

5. Aditya-L1: a 110-day cruise to 1.5 million km

Background. Aditya-L1, India's first solar observatory, was launched by PSLV-C57 on 2 September 2023 into a 235.6×19502.7 km orbit. Its destination was a halo orbit about the Sun–Earth L_1 point, about 1.5 million km from the Earth towards the Sun, from where it can watch the Sun without interruption. Four Earth-bound burns raised the apogee to 121 973 km; ISRO describes the strategy as one that minimised Δv while “restricting the number of perigee passes”, to limit the time spent in the Van Allen radiation belts. The trans- L_1 injection (TL1I) of 19 September sent it away; it left the Earth's sphere of influence on 30 September, and after a cruise of about 110 days it was inserted into its halo orbit on 6 January 2024.

date (2023)	event	orbit after [km]
2 Sep	PSLV-C57 injection	235.6×19502.7
3 Sep	Earth-bound burn 1	245×22459
5 Sep	Earth-bound burn 2	282×40225
10 Sep	Earth-bound burn 3	296×71767
15 Sep	Earth-bound burn 4	256×121973

date (2023)	event	orbit after [km]
19 Sep	trans- L_1 injection	departure towards L_1
30 Sep	leaves the Earth's sphere of influence	—
6 Jan 2024	halo-orbit insertion	halo, period ≈ 178 d

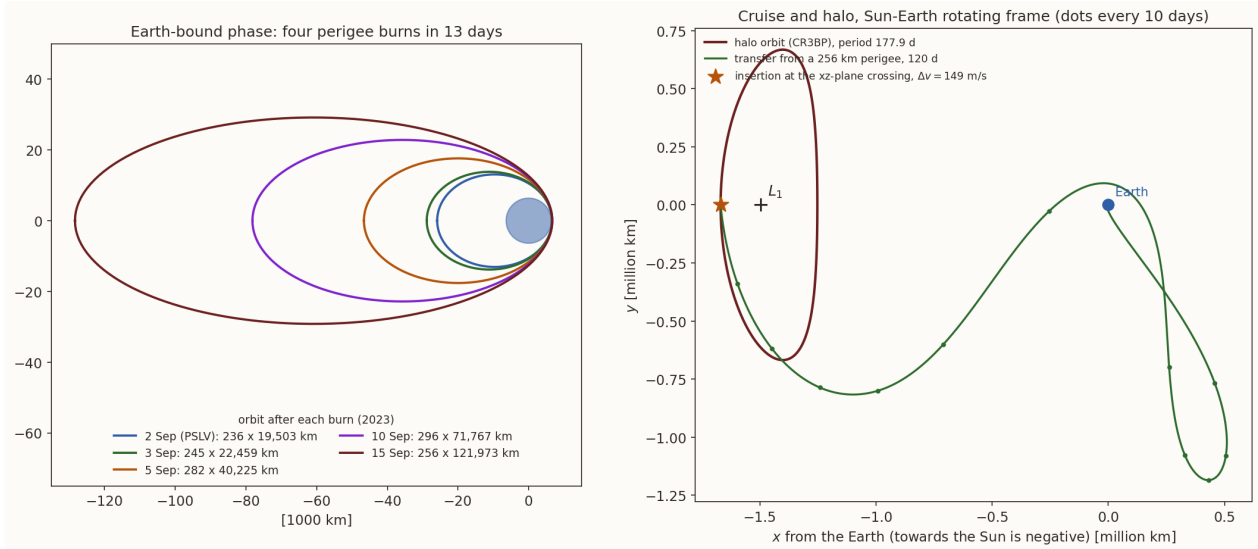


Figure 5: Aditya-L1, reconstructed. Left: the Earth-bound orbits, to scale. Right: the transfer and the halo orbit in the Sun–Earth rotating frame (see Problem 6). [Script](#).

(a) Find the Δv of the four Earth-bound burns, and the period of each orbit. Using the dates, estimate how many perigee passes the spacecraft made before TL1I. Compare with Mangalyaan, which made six burns in eleven days. Why does the number of perigee passes matter?

Answer. The four burns cost 104.2, 372.7, 268.5 and 165.2 m/s: 911 m/s in all. The periods are 5.7, 6.5, 12.0, 24.1 and 48.5 h for the five orbits. Counting days between burns divided by the period: about 4 passes in the launch orbit (1 day), 7 in the second (2 days), 10 in the third (5 days), 5 in the fourth (5 days) and 2 in the last (4 days): roughly 28 passes.

Every perigee pass at a few hundred km means two crossings of the inner Van Allen belt, where the energetic protons degrade solar cells and electronics. Aditya-L1 did the work in four burns, the second of them large (373 m/s), so that the apogee grew quickly and the number of passes stayed small. The price of a large burn is a longer firing and a larger finite-burn loss (Problem 1(c)).

(b) **TL1I as a two-body problem.** Find the Δv at the 256 km perigee that raises the apogee to 1.5×10^6 km, and the Δv that would escape the Earth altogether. On the two-body ellipse, how long does the climb to 1.5×10^6 km take, and how long to reach the edge of the Earth's sphere of influence, $r_{\text{SOI}} = a_{\oplus}(\mu_{\oplus}/\mu_{\odot})^{2/5}$? Compare with the dates.

Answer. At $r_p = 6634.1$ km the $256 \times 121\,973$ km orbit has $v_p = 10.689$ km/s. An ellipse with apogee radius 1.5×10^6 km has $a = 753\,317$ km and $v_p = 10.938$ km/s, so $\text{TL1I} \approx 249$ m/s; escape needs $\sqrt{2\mu/r_p} = 10.962$ km/s, i.e. 273 m/s. The trip to L_1 is nearly an escape: only 24 m/s separate the two.

The half-period of that ellipse is $\pi\sqrt{a^3/\mu} = 37.7$ days. The sphere of influence has $r_{\text{SOI}} = 1.496 \times 10^8 (3.0 \times 10^{-6})^{0.4} \approx 925\,000$ km. To find the time to reach it, use the eccentric anomaly: $e = 1 - r_p/a = 0.9912$, $\cos E = (1 - r/a)/e$ gives E , and Kepler's equation $nt = E - e \sin E$ gives $t = 10.0$ days. ISRO announced the exit from the sphere of influence on 30 September, 11 days after TL1I: the two-body model is good here, close to the Earth.

But the cruise to the halo orbit took about 110 days, three times the two-body 37.7. Near L_1 the Sun's pull is as strong as the Earth's, by definition, and the two-body ellipse is not the right model there. Problem 6 does it properly.

6. Aditya-L1 in the three-body problem (*after CR3BP*)

Background. In the Sun–(Earth+Moon) circular restricted three-body problem, with $\mu = \frac{\mu_\oplus + \mu_M}{\mu_\odot + \mu_\oplus + \mu_M} = 3.0404 \times 10^{-6}$, length unit 1 AU and time unit $1/n$ (a year is 2π), Aditya-L1’s halo orbit is a periodic orbit about L_1 . ISRO gave its target semi-axes as $A_x = 209\,200$ km, $A_y = 663\,200$ km and $A_z = 120\,000$ km, and its period as 177.86 days. The halo-orbit insertion (HOI) was made as the spacecraft crossed the xz plane of the rotating frame: the burn cancelled the x and z components of the velocity and set the y component the halo orbit needs. Two trajectory-correction manoeuvres (5 October and 14 December 2023) kept the spacecraft on course for that crossing.

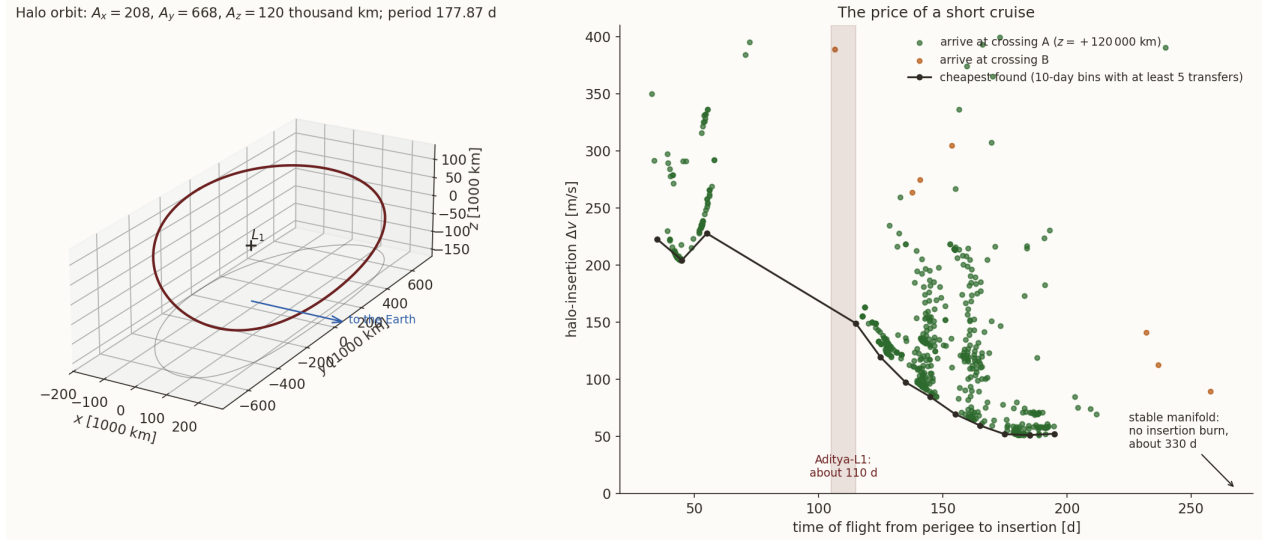


Figure 6: Aditya-L1’s halo orbit in the CR3BP, computed by differential correction with $A_z = 120\,000$ km, and the cost of insertion as a function of the transfer time. [Script](#).

(a) **Units and L_1 .** Find the time and speed units of the problem. L_1 lies on the Sun–Earth line at a distance γ (in AU) from the Earth, where the gravity of the two bodies and the centrifugal term balance:

$$x - \frac{(1-\mu)(x+\mu)}{|x+\mu|^3} - \frac{\mu(x-1+\mu)}{|x-1+\mu|^3} = 0, \quad x = 1 - \mu - \gamma.$$

Solve it numerically, and compare with the Hill estimate $\gamma \approx (\mu/3)^{1/3}$.

Answer. The time unit is $1/n$, with n the Earth’s mean motion: $1/n = 58.13$ days, so a year is 2π units. The speed unit is $\text{AU} \cdot n = 29.785$ km/s, the Earth’s orbital speed.

The equation has one root between the Sun and the Earth. Newton’s method from the Hill value gives $\gamma = 0.0100110$ AU, i.e. 1.4976×10^6 km from the Earth: the “1.5 million km” of ISRO’s descriptions. The Hill estimate is $(\mu/3)^{1/3} = 0.0100447$ AU = 1.5027×10^6 km, only 0.34% too far: the Sun’s and the Earth’s pulls balance where the Earth’s tidal field is as strong as its direct one, and the Hill approximation keeps exactly those two terms.

(b) **Can the two-body TL1I reach L_1 ?** The Jacobi constant is $C = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - v^2$, with v the speed in the rotating frame; a spacecraft can never enter regions where $x^2 + y^2 + 2(1-\mu)/r_1 + 2\mu/r_2 < C$. Find C at L_1 , on the halo orbit, and at a 256 km perigee with the two-body TL1I speed of Problem 5(b), 10.938 km/s (inertial). What perigee speed is needed to reach the halo orbit’s energy?

Answer. At L_1 the speed is zero: $C_{L_1} = 3.000\,898$. On the halo orbit, $C_{\text{halo}} = 3.000\,827$, slightly smaller: the halo orbit has a little more energy than L_1 itself, as it must, since it moves round L_1 .

At the perigee, convert the inertial velocity to the rotating frame, $\mathbf{v}_{\text{rot}} = \mathbf{v}_{\text{in}} - \hat{\mathbf{z}} \times \mathbf{r}$. With 10.938 km/s, $C = 3.002\,280$. That is **larger** than C_{L_1} : the zero-velocity surface is closed at the L_1 neck, and the spacecraft can never get there. The two-body estimate of Problem 5(b) is not enough.

Solving $C(v) = C_{\text{halo}}$ for the perigee speed gives 10.997 km/s; to open the L_1 neck at all needs 10.994 km/s. Both are a little *above* the two-body escape speed at that perigee, 10.962 km/s: in the rotating frame the Sun's pull and the frame's rotation take back some of the energy. So the TL1I must be about $10.9968 - 10.6893 = 307$ m/s, not 249 m/s. Every transfer found in part (e) has a perigee speed between 10.997 and 11.003 km/s.

(c) The halo orbit. A differential-correction computation (in the script) starting on the xz plane at $z = 120\,000$ km finds a periodic orbit with period 177.87 days and semi-axes $A_x = 208\,270$ km, $A_y = 667\,997$ km. Compare with ISRO's target. Then explain why its period is so close to half a year: linearise about L_1 , with $c_2 = \mu/\gamma^3 + (1 - \mu)/(1 - \gamma)^3 = 4.0611$, to find the in-plane frequency ω_p and the out-of-plane frequency $\omega_v = \sqrt{c_2}$, and their periods.

Answer. The computed orbit agrees with ISRO's target ($A_x = 209\,200$, $A_y = 663\,200$, $A_z = 120\,000$ km, 177.86 d) to 0.4% , 0.7% , exactly (that was the input), and 0.004% . A halo orbit is fixed by one amplitude; the other two, and the period, follow.

The linearised equations about L_1 are

$$\ddot{\xi} - 2\dot{\eta} - (1 + 2c_2)\xi = 0, \quad \ddot{\eta} + 2\dot{\xi} + (c_2 - 1)\eta = 0, \quad \ddot{\zeta} + c_2\zeta = 0.$$

The in-plane pair has the oscillatory root $\omega_p^2 = \frac{1}{2}(2 - c_2 + \sqrt{9c_2^2 - 8c_2})$, so $\omega_p = 2.0865$ and the period is $2\pi/\omega_p$ units = 175.06 days. The vertical frequency is $\omega_v = \sqrt{c_2} = 2.0152$, period 181.25 days. The two are not equal, so the linear motion is a Lissajous figure, not a closed orbit. A *halo* orbit is the large-amplitude orbit for which the nonlinear terms make the two periods equal; it lands between them, at 177.9 days. Every L_1 halo orbit of this size has a period of about half a year.

The linear theory also gives $A_y/A_x = k = (\omega_p^2 + 1 + 2c_2)/(2\omega_p) = 3.23$; the computed orbit has $667\,997/208\,270 = 3.21$.

(d) Why station-keeping is needed. The monodromy matrix of the halo orbit (its state transition matrix over one period) has a real eigenvalue $\lambda = 1735$. What is the e-folding time of a small error? ISRO's station-keeping burns came 47 , 106 and 25 days apart. Is that consistent?

Answer. An error along the unstable direction is multiplied by 1735 each period, so it grows as $e^{t/\tau}$ with $\tau = T/\ln \lambda = 177.87/7.459 = 23.8$ days. (The linear estimate at L_1 , $1/\lambda_{\text{lin}}$ with $\lambda_{\text{lin}} = 2.533$, gives 23.0 days.) A 1 cm/s error grows to about 44 cm/s in three months, and the position error grows with it. So the orbit must be corrected every few e-folding times, which is what the spacing of 25 to 106 days does; each correction is small because the error is caught early. The same instability is what makes the transfer of part (e) possible: the halo orbit has a *stable* manifold too, a family of trajectories that approach it with no burn at all.

(e) The price of a short cruise. The right panel of Figure 6 shows transfers from a 256 km perigee to the crossing A of the halo orbit (the point $z = +120\,000$ km on the xz plane), found by integrating backwards from the halo with a trial insertion Δv and keeping the trajectories whose first perigee is at 256 km. The cheapest found are about 205 m/s for a 45 -day transfer, 150 m/s at 115 days, 85 m/s at 145 days and 51 m/s at 185 days. Following the halo orbit's stable manifold backwards from 200 km off the orbit reaches a perigee of 64 km after 327 days, with no insertion burn at all.

- (i) Explain the trend. (ii) For a spacecraft of about 1500 kg, what does the fastest of these transfers cost in propellant, compared with the manifold? (iii) ISRO describes Aditya-L1's insertion, after a cruise of about 110 days, as a short firing of its control engines: far less than the 150 m/s this model needs at 115 days. What does the model leave out that could account for the difference?

Answer.

- (i) A trajectory on the stable manifold reaches the halo orbit with exactly the halo orbit's velocity, so no burn is needed. But it approaches asymptotically: the distance to the orbit shrinks by a factor e every 23.8 days (part (d)), so getting from a few hundred km off the orbit to "on it" takes months. To arrive sooner, the spacecraft must leave the manifold and arrive with a velocity that differs from the halo orbit's; the insertion burn is that difference, and the sooner the arrival, the larger it is. The fastest transfers, about 40 days, are close to the two-body ellipse of Problem 5(b) (37.7 days), which arrives with no regard for the halo orbit's velocity and pays about 200 m/s for it. Time and propellant trade against each other, as in the phasing problems of the *Orbital Maneuvers* deck, but here the trade is set by the instability of the halo orbit.
- (ii) The departure burn is about 307 m/s for every one of these transfers (part (b)): their perigee speeds all lie between 10.997 and 11.003 km/s. So the difference is in the insertion. An extra 205 m/s on 1500 kg costs $1500(1 - e^{-0.205/3.040}) = 98$ kg of propellant, about 7% of the spacecraft. The manifold costs nothing at insertion but takes three times as long, and its long, slow approach leaves a long time for errors to grow.
- (iii) Several things, and none of them can be checked with this model alone:
 - the real problem has the Moon's gravity and the eccentricity of the Earth's orbit, which the CR3BP leaves out, and both move the manifolds;
 - the real transfer was shaped over the whole cruise by two trajectory-correction burns (on 5 October and 14 December), so not all the matching was left to the insertion;
 - the real insertion point and the phase of the halo orbit at arrival were design choices; here both are fixed (at crossing A of one particular halo orbit), and the departure perigee is constrained only in altitude;
 - the search here is local (a coarse grid refined by Nelder–Mead), so cheaper 110-day transfers may well exist in this model too.

The lesson that does carry over is the shape of the curve: a slow arrival along, or near, the stable manifold is what makes the insertion small.

Sources and caveats

The mission figures are ISRO's published values, mostly as collected in the Wikipedia articles, and checked against ISRO's own pages where these exist. Orbits are rounded as published.

- **Chandrayaan-1** (Problem 1): orbits, burn durations, the 1380 kg launch mass and 675 kg in lunar orbit from the [Wikipedia article](#) and ISRO's status reports of October–November 2008 (for example [this one](#)). The perigee of the lunar transfer orbit was not published; 465 km is assumed.
- **Chandrayaan-2** (Problem 2): orbits and burn durations from the [Wikipedia article](#); the trans-lunar target orbit from ISRO's [optimised mission plan](#).
- **Chandrayaan-3** (Problem 3): the lunar orbits from ISRO's [insertion update](#) and the [Wikipedia article](#); the propulsion module's dry mass (448.62 kg with pressurant) from the [LVM3-M4 brochure](#); its return from ISRO's [report of 4 December 2023](#).
- **Mangalyaan** (Problem 4): orbits, masses and the Mars orbit from the [Wikipedia article](#); the planned 130 m/s of the short burn from ISRO's statement of 11 November 2013. Planet positions are from the [plan94](#) analytical ephemeris of ERFA, accurate to a few thousand km, which is ample for a Lambert arc.
- **Aditya-L1** (Problems 5 and 6): orbits and dates from ISRO's [mission page](#); the parking orbit, the halo semi-axes, the insertion procedure and the 110-day cruise from ISRO's [halo-insertion release](#); the period and the station-keeping dates from [this ISRO update](#).
- **Models.** The I_{sp} of 310 s is assumed for all five spacecraft. The lunar arrivals are coplanar patched conics, and the actual arrivals, shaped by the inclination of the lunar orbit and by the Moon's pull on the way in, differ by tens of m/s. The CR3BP of Problem 6 lumps the Earth and Moon together and leaves out the eccentricity of the Earth's orbit, so it reproduces

the character of Aditya-L1's transfer, not its exact trajectory.

- **Scripts.** `codes/_imcommon.py` (models), `codes/fig-im-*.py` (figures), `codes/aditya-hoi-scan.py` and `codes/aditya-hoi-refine.py` (the transfers of Problem 6, saved in `codes/data/`), and `codes/indian-missions-verify.py` (every number on this page). Run them from the `sfm-slides` directory.