Ground Tracks

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October 31, 2026

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Where we are going

A satellite moves on a fixed ellipse in inertial space, but we watch it from a rotating Earth. The **ground track** is the curve that its sub-satellite point

traces on a map, and almost every question about what a satellite *sees*, when it passes over a station, or when it returns to the same place, is a question about that curve. This note turns six orbital elements into the curve (with an interactive simulator to try any orbit), looks at the standard families of orbit (low, polar, sun-synchronous, geosynchronous, Molniya), and ends with what the curve is *for*.

> What you should be able to answer at the end
>
> (1) What are the steps from orbital elements to latitude and longitude? (2) Why is the maximum latitude of a track equal to the inclination? (3) Why does the track of a low orbit move about 24° west on every orbit? (4) What is a repeat ground track, and how do we find the altitude that gives one? (5) Which ground-track property is each of the following applications exploiting: Earth observation, communications, navigation, ground-station contact, launch?

The required background is the two-body problem, Kepler's equation, the orbital elements and the rotation matrices between the perifocal, inertial and Earth-fixed frames. The Earth's rotation rate, the sidereal day and the Greenwich angle θ_G are explained in the time note, which comes first. The secular effect of J_2 on Ω, ω and M is quoted from the note Beyond the Point Mass. Ground *stations*, and the angles and ranges measured from them, are the subject of the topocentric note.

Symbols and constants shared by all the notes are collected on the notation page; the study plan gives the order in which to read the notes.

1 What is a ground track?

The **ground track** of a satellite is the curve that the *sub-satellite point* traces on the surface of the Earth as the Earth turns beneath the orbit. The sub-satellite point is the point where the line from the satellite to the Earth's centre (or, more precisely, the local vertical) pierces the surface. It is described by a latitude φ and an east longitude λ.

The ground track answers the questions "what does the satellite see?", "when is it over my station?" and "when will it return to the same place?". Almost every application (Section 5) is a statement about the ground track.

2 The method, step by step

The orbit is a fixed ellipse in inertial space. The ground track appears because the Earth turns beneath it. That is the whole idea, and it separates the computation into two parts: find the position in an inertial frame (steps 2 to 4 of Figure 1), then express it in the Earth-fixed frame (steps 5 and 6).

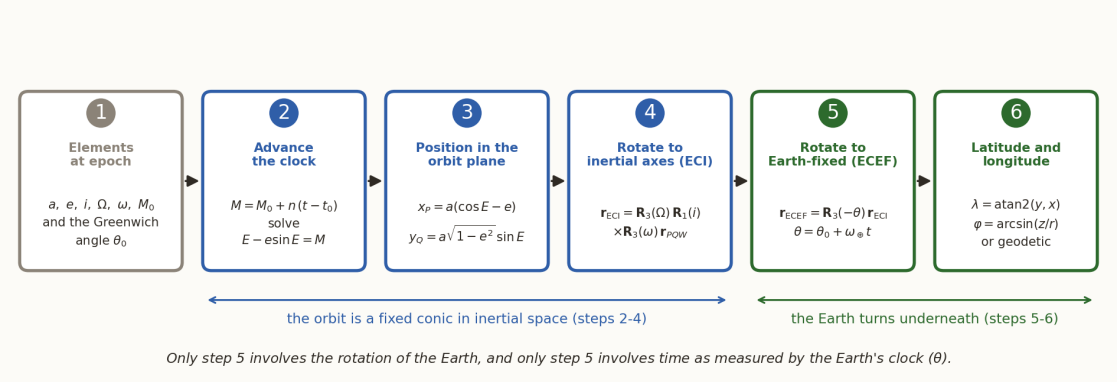


Figure 1: The six steps from orbital elements to latitude and longitude. Steps 2 to 4 describe a conic in inertial space and do not involve the Earth's rotation. Step 5 is the only place where the Earth's rotation, i.e. the Earth's clock, enters. *Script: fig-gt-method.py.*

Step 1: the data. The six elements $(a, e, i, \Omega, \omega, M_0)$ at the epoch t_0, and the Greenwich sidereal angle θ_0 at the epoch, from the sidereal-time formula of the time note.

Step 2: advance the clock. With M_0 the mean anomaly at the epoch t_0, the mean anomaly at time t is $M = M_0 + n(t - t_0)$, $n = \sqrt{\mu/a^3}$, and Kepler's equation $E - e \sin E = M$ is solved for E (Newton's method).

Step 3: position in the orbit plane (perifocal frame, $\hat{\mathbf{p}}$ towards perigee):

$$x_P = a(\cos E - e), \qquad y_Q = a\sqrt{1 - e^2}\,\sin E, \qquad z_W = 0. \tag{1}$$

Step 4: to inertial axes. Three rotations bring the orbit plane to its place: about the normal by ω (perigee direction), about the line of nodes by i (tilt), about the polar axis by Ω (node). With $\mathbf{R}_1$ and $\mathbf{R}_3$ the *active* rotation matrices of the perifocal note, which turn a vector by θ about x and z,

$$\mathbf{r}_{\mathrm{ECI}} = \mathbf{R}_3(\Omega)\,\mathbf{R}_1(i)\,\mathbf{R}_3(\omega)\,\mathbf{r}_{PQW}, \qquad \mathbf{R}_3(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{R}_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}. \tag{2}$$

(Some books, Curtis among them, use *passive* matrices that rotate the frame instead; each is the transpose of the active one, $R_3^{\text{passive}}(\theta) = \mathbf{R}_3(-\theta)$, so the same product reads $R_3(-\Omega)R_1(-i)R_3(-\omega)$ in their notation.)

Step 5: to Earth-fixed axes. The Earth-fixed frame is the inertial frame turned about the polar axis through the Greenwich angle $\theta(t) = \theta_0 + \omega_\oplus(t - t_0)$, with $\omega_\oplus$ from the Earth's rotation rate. A vector fixed in space therefore appears turned *backwards* by θ in Earth-fixed components:

$$\mathbf{r}_{\text{ECEF}} = \mathbf{R}_3(-\theta(t))\,\mathbf{r}_{\text{ECI}}. \tag{3}$$

Step 6: latitude and longitude. With $(x, y, z) = \mathbf{r}_{\text{ECEF}}$ and $r = |\mathbf{r}|$, the *geocentric* coordinates are

$$\lambda = \operatorname{atan2}(y, x), \qquad \varphi_{\text{gc}} = \arcsin\frac{z}{r}. \tag{4}$$

Maps and GPS use the *geodetic* latitude, measured from the local normal to the WGS-84 ellipsoid (flattening $f_\oplus = 1/298.257$). It is obtained by a short fixed-point iteration, with $p = \sqrt{x^2 + y^2}$, $e_E^2 = f_\oplus(2 - f_\oplus)$:

$$\varphi \leftarrow \operatorname{atan2}\!\left(z,\; p\left(1 - e_E^2\,\frac{N}{N+h}\right)\right), \quad N = \frac{R_\oplus}{\sqrt{1 - e_E^2\sin^2\varphi}}, \quad h = \frac{p}{\cos\varphi} - N, \tag{5}$$

started from $\varphi = \operatorname{atan2}(z,\, p(1 - e_E^2))$; six passes converge to well below a millimetre. The geodetic and geocentric latitudes differ by at most 0.19° (about 21 km on the ground), so for a picture at the scale of a continent either will do, but for pointing an antenna it matters.

The six steps are implemented in _gt.py, which draws every ground track in this note.

> **Where the clocks of the time note come in**
>
> Time enters twice: through $M = M_0 + n\,t$ and through $\theta = \theta_0 + \omega_\oplus t$. A timing error of δt moves the satellite along its orbit by $v\,\delta t$ (step 2) *and* turns the ECEF frame by $\omega_\oplus\delta t$ (step 5). For a 500 km orbit the first is 7.6 km per second of error. The second is 0.465 km per second at the equator, and it is the part that depends on UT1 rather than UTC (time note).

The animation shows the two views together for an orbit of 60° inclination and 1200 km altitude. On the left, the orbit plane (blue ring) does not move; the globe turns beneath it, and the red curve is painted on the *rotating* surface. On the right is the same curve on a map.

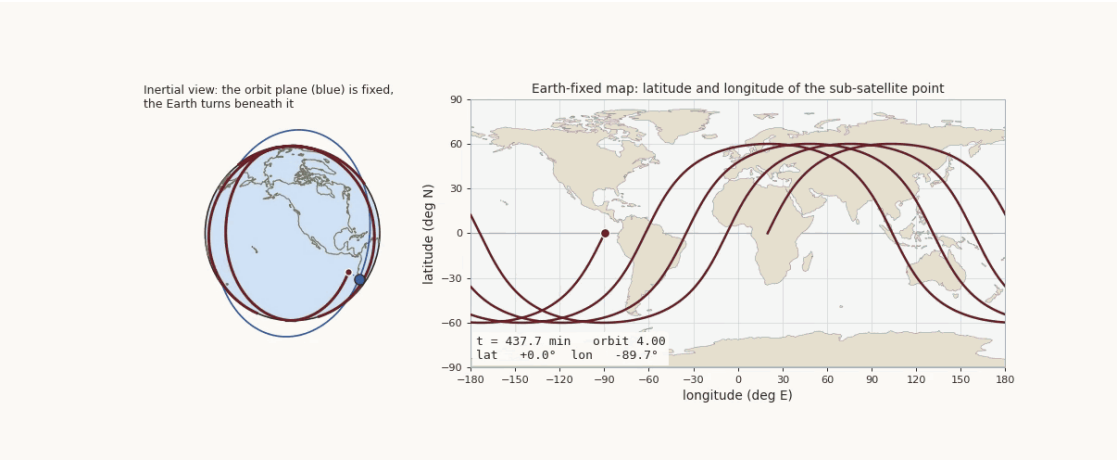


Figure 2: The orbit is fixed, the Earth turns, and the ground track appears ($i = 60°$, $h = 1200$ km, four orbits). *Script: anim-gt-build.py.*

2.1 A closed form for circular orbits

For a circular orbit ($e = 0$) the six steps collapse into a formula that we can read. Use the argument of latitude $u = \omega + f$ (the angle from the ascending node to the satellite, measured in the orbit plane). Steps 3 and 4 give the inertial unit vector

$$\hat{\mathbf{r}}_{\mathrm{ECI}} = (\cos\Omega\cos u - \sin\Omega\cos i\sin u,\ \sin\Omega\cos u + \cos\Omega\cos i\sin u,\ \sin i\sin u).$$

The last component is $\sin\varphi$ directly, and the first two can be combined as $x + \mathrm{i}y = e^{\mathrm{i}\Omega}(\cos u + \mathrm{i}\cos i\sin u)$. Hence (Figure 3)

$$\boxed{\sin\varphi = \sin i\,\sin u, \qquad \tan(\alpha - \Omega) = \cos i\,\tan u, \qquad \lambda = \alpha - \theta(t)} \tag{6}$$

where α is the right ascension of the satellite. The first two relations are the standard relations of the right spherical triangle N–F–S of Figure 3 (hypotenuse u, angle i at N, sides $\alpha - \Omega$ and φ). The third turns inertial right ascension into Earth-fixed longitude. Nothing in the derivation used $e = 0$: the three relations hold on any orbit, with $u = \omega + f$. What is special about the circular orbit is only that u then grows uniformly in time, which is what the properties below use.

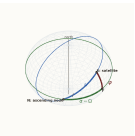


Figure 3: The right spherical triangle behind Equation 6: N is the ascending node, S the satellite, F its foot on the equator. The hypotenuse is u, the angle at N is i, the sides are $\alpha - \Omega$ (along the equator) and φ (along the meridian). *Script: fig-gt-method.py.*

Reading Equation 6 gives the first properties of every circular ground track.

1. **The largest latitude equals the inclination.** $\sin\varphi \le \sin i$, so $|\varphi|_{\max} = i$ for $i \le 90°$ and $180° - i$ for a retrograde orbit. The track lies between these parallels and touches them once per orbit (Figure 5, dashed lines). A polar orbit reaches the poles; an equatorial orbit never leaves the equator.

2. **The track shifts west by $\omega_\oplus T$ on every orbit.** After one period u has increased by 2π but θ by $\omega_\oplus T$, so the next equator crossing is displaced by $-\omega_\oplus T$ in longitude:

$$\Delta\lambda_{\text{orbit}} = -\omega_\oplus T = -360^\circ \frac{T}{T_{\text{sid}}} \quad (\text{with } J_2 : \ -(\omega_\oplus - \dot\Omega)\, T_\Omega), \tag{7}$$

where T_Ω is the nodal period, the time between successive ascending nodes (defined precisely in Section 3.6). For $T = 92.97$ min (the ISS) this is 23.3°, i.e. 2594 km at the equator; for a 650 km orbit 24.5° (2727 km). The number of orbits per day is $86\,400/T$: 15.49 for the ISS.

3. **The heading at the equator is not the inclination.** At the ascending node $u = 0$, $\dot\varphi = n \sin i$ and $\dot\lambda = n\cos i - \omega_\oplus$, so the track crosses the equator at an angle β (measured from east) with

$$\tan\beta = \frac{n \sin i}{n\cos i - \omega_\oplus}. \tag{8}$$

For the ISS ($\omega_\oplus/n = 0.0647$, $i = 51.64^\circ$) this gives $\beta = 54.7^\circ$, steeper than the inertial 51.64°: the surface is moving eastward under the satellite. For a retrograde orbit the Earth's rotation *adds* to the westward motion, and for a geostationary orbit ($n = \omega_\oplus$, $i = 0$) the numerator and the denominator both vanish and the track collapses to a point.

4. **Small inclination at the geosynchronous radius gives a figure 8.** If $n = \omega_\oplus$ (the orbit is geosynchronous) then $\lambda = \lambda_\Omega + [\operatorname{atan2}(\cos i \sin u, \cos u) - u]$, where $\lambda_\Omega = \Omega - \theta + u$ is constant (because $\dot u = \dot\theta$) and is the geographic longitude of the node, and, for small i, $\operatorname{atan2}(\cos i \sin u, \cos u) - u \approx -\frac{i^2}{4}\sin 2u$. The track is

$$\varphi = i \sin u, \qquad \lambda - \lambda_\Omega \approx -\frac{i^2}{4}\sin 2u, \tag{9}$$

a figure 8 of height $2i$ and width $i^2/2$, traversed once per sidereal day (Figure 8, right, dotted: Equation 9). For $i = 29^\circ$ the exact longitude excursion is $\pm 3.84^\circ$ and Equation 9 gives $\pm 3.67^\circ$; for $i = 10^\circ$, $\pm 0.439^\circ$ against $\pm 0.436^\circ$.

2.2 The effect of J_2, and mean against osculating elements

Over many orbits the equatorial bulge (see Beyond the Point Mass) turns the orbit plane and rotates the perigee, and changes the mean motion. The secular rates, with $p = a(1 - e^2)$, are

$$\dot\Omega = -\tfrac{3}{2} n J_2 \Big(\frac{R_\oplus}{p}\Big)^2 \cos i, \quad \dot\omega = \tfrac{3}{4} n J_2 \Big(\frac{R_\oplus}{p}\Big)^2 (5\cos^2 i - 1), \quad \dot M = n\Big[1 + \tfrac{3}{4} J_2 \Big(\frac{R_\oplus}{p}\Big)^2 \sqrt{1-e^2}\,(3\cos^2 i - 1)\Big] \tag{10}$$

To include J_2 in the ground track, replace M, Ω, ω in steps 2 to 4 by $M_0 + \dot{M}t$, $\Omega_0 + \dot{\Omega}t$, $\omega_0 + \dot{\omega}t$. The node drift changes Equation 7 as shown, and it is what makes sun-synchronous orbits possible (Section 3.3).

There is one trap. The rates of Equation 10 hold for **mean** elements: the elements averaged over an orbit. The **osculating** elements computed from an instantaneous position and velocity oscillate around them; in a the amplitude is about $\frac{3}{2}J_2R_\oplus^2/a$, roughly 10 km for a low orbit. Figure 4 makes the point. We integrated the equations of motion with the J_2 acceleration for three days (DOP853, relative tolerance 10^{-11}) for an 800 km orbit at $i = 98.6°$, and compared with the analytic ground track. If the osculating elements at $t = 0$ are used as mean elements, the two tracks are 76 km apart after one orbit, 1057 km after a day and 3256 km after three days: the wrong semi-major axis (osculating 7178.1 km, mean 7187.2 km) makes the satellite run early. If the mean elements are *fitted* to the integration, the analytic track follows the integrated one to 0.48 km rms and 0.69 km at most over three days. The lesson is the standard one of mission analysis: **use mean elements to describe orbits over long periods, and never mix them with osculating ones.**

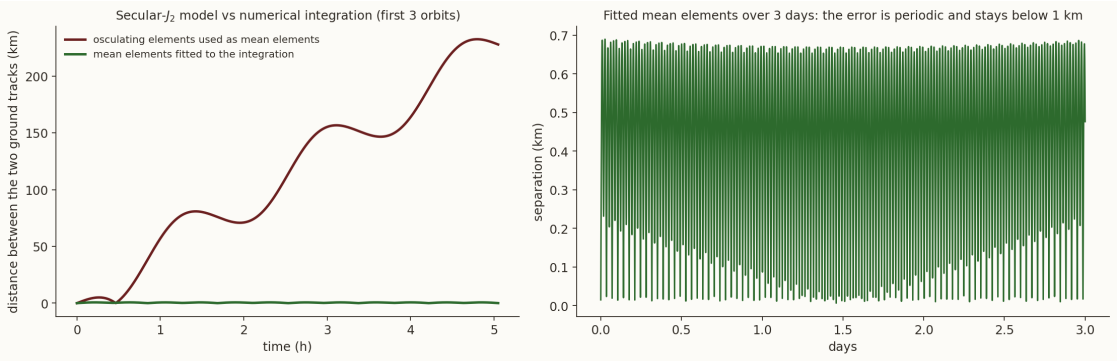


Figure 4: **Left:** distance between the analytic (secular J_2) and numerically integrated ground tracks during the first three orbits, when the osculating elements are used as if they were mean elements (red) and when mean elements are fitted (green). **Right:** the fitted case over three days. *Script: fig-gt-method.py.*

3 The standard ground tracks

3.1 Low Earth orbit

Figure 5 shows one day of three low orbits: a near-equatorial one (like India's Astrosat, $i = 6°$, 650 km), one at 28.5° (the latitude of Cape Canaveral, the lowest inclination reachable from there without a plane change), and the International Space Station (51.6°, 420 km). All three follow Equation 6: a sinusoid-like curve confined to the band $|\varphi| \leq i$, shifted west by 23 to 25° on every orbit. In a day the ISS completes 15.5 orbits, with successive tracks

about 2600 km apart at the equator; over a few days it passes over every point between 51.6°N and 51.6°S, and it never sees the poles.

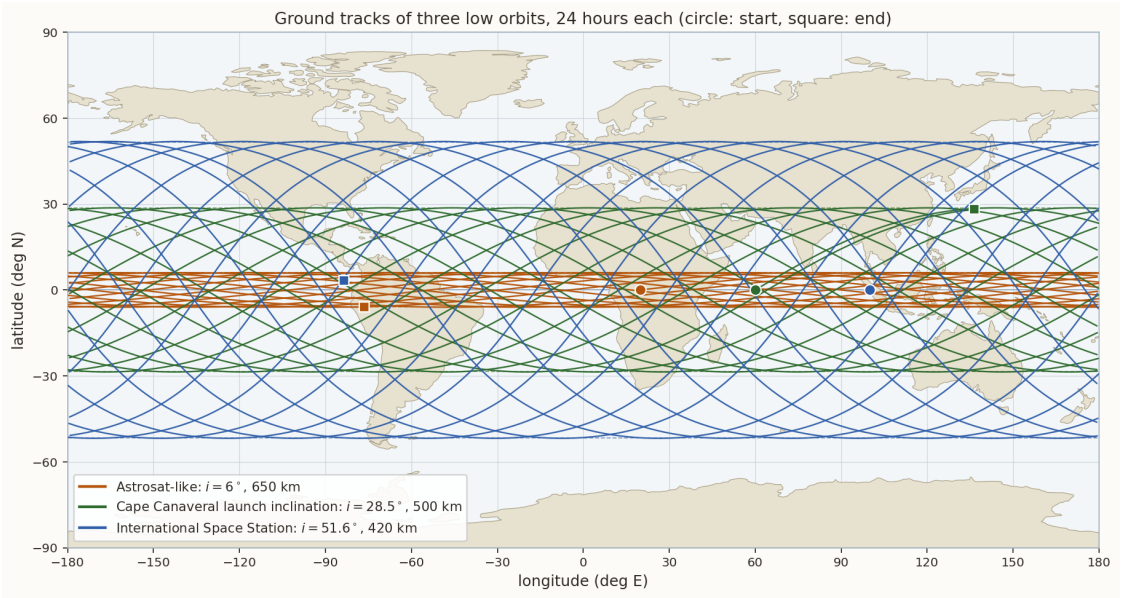


Figure 5: One day of three low orbits (circle: start, square: end). The dashed lines mark $\pm i$, the highest latitudes reached. *Script: fig-gt-tracks.py.*

3.2 Polar orbit

A polar orbit ($i = 90°$) passes over both poles on every revolution and, as the Earth turns underneath, eventually over every point on the globe (Figure 6). At 800 km the period is 100.9 min, there are 14.3 orbits per day, and successive tracks are 25.3° (2800 km) apart at the equator. All tracks cross at the poles, so coverage is dense at high latitudes and sparsest at the equator. With J_2 a polar orbit has $\dot{\Omega} = 0$: its plane stays fixed in inertial space, so relative to the Sun it turns once a year.

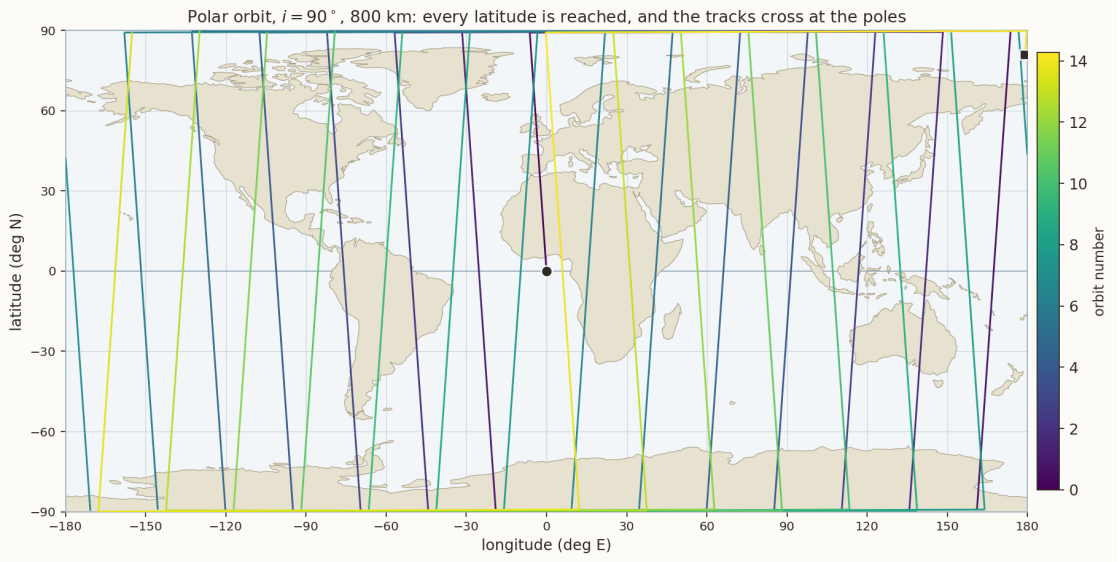


Figure 6: A polar orbit at 800 km: one day, coloured by orbit number. *Script: fig-gt-tracks.py.*

3.3 Sun-synchronous orbit

A **sun-synchronous orbit** (SSO) is not an orbit that follows the Sun. It is an orbit whose *plane* turns at exactly the rate at which the Sun's right ascension increases,

$$\dot{\Omega} = \frac{2\pi}{365.2422\ \mathrm{d}} = +0.98565^\circ/\mathrm{day}, \tag{11}$$

so that the angle between the orbit plane and the Earth–Sun line never changes. The satellite then crosses the equator at the same *local mean solar time* on every pass for the whole mission, which means the same sun elevation, the same shadow lengths and an unchanging power and thermal environment.

Equation 10 gives $\dot{\Omega} \propto -\cos i$, and Equation 11 needs $\dot{\Omega} > 0$, so **every sun-synchronous orbit is retrograde**. Solving $\dot{\Omega} = 0.98565^\circ/\mathrm{day}$ for the inclination gives, for circular orbits, $i = 98.60^\circ$ at 800 km and $i = 98.19^\circ$ at 700 km. The highest latitude is then $180^\circ - i = 81.4^\circ$, so an SSO covers the whole globe except for small polar caps. The right panel of Figure 7 shows the payoff: the local time of the ascending node stays constant for a year, whereas a polar orbit (fixed plane) drifts through 24 h in a year (-1.97 h per 30 days) and the ISS orbit through 24 h in about two months, 61 days (-11.9 h per 30 days).

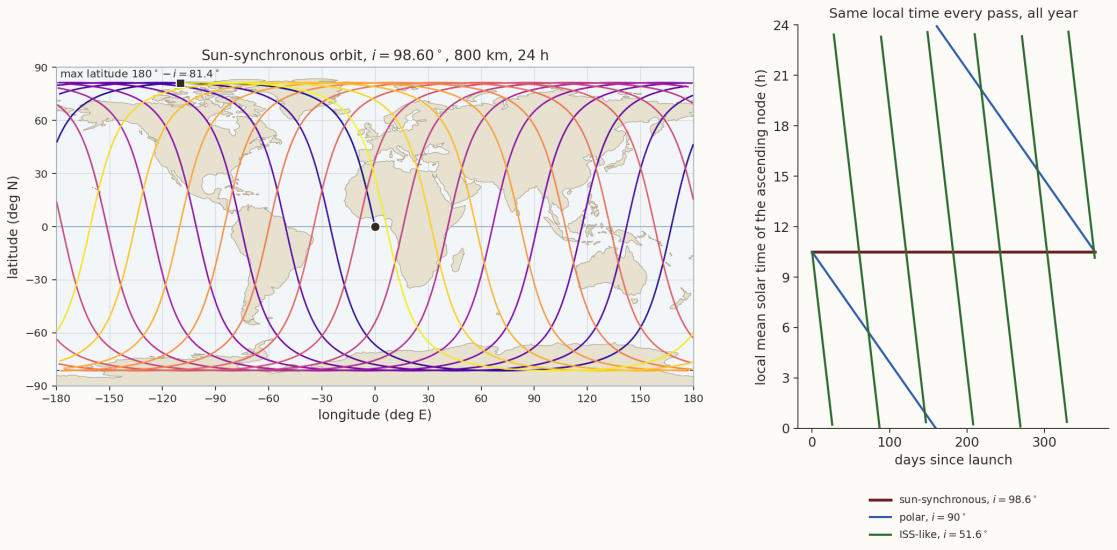


Figure 7: **Left:** one day of a sun-synchronous orbit at 800 km ($i = 98.60°$). **Right:** local mean solar time of the ascending node against time, for a sun-synchronous, a polar and an ISS-like orbit; all start at 10.5 h. *Script: fig-gt-tracks.py*.

3.4 Geostationary and geosynchronous orbits

At $a = 42\,164$ km ($T =$ one sidereal day) and $i = 0$ the sub-satellite point is *fixed*: the geostationary orbit is the only orbit whose ground track is a point. Its ground track is a single dot, and a single dot is exactly what a communications or weather satellite wants: a ground antenna does not have to move. At this altitude a satellite sees 42% of the surface (Section 5), so three satellites cover the equatorial and mid-latitude world.

If the same orbit is inclined ($i > 0$) it is **geosynchronous** but no longer stationary: the sub-satellite point traces the figure 8 of Equation 9, once a day (Figure 8). India's regional navigation system NavIC uses exactly this combination: three geostationary satellites (at 32.5°E, 83°E and 131.5°E) and four inclined geosynchronous ones ($i = 29°$) whose figures of eight are centred at 55°E and 111.75°E, so that at any moment several satellites are high in the sky over India. Historically, Syncom 2 (1963), the first geosynchronous satellite, had an inclination of 33° and drew a figure 8 before Syncom 3 (1964) became the first geostationary one.

History. The first Earth-observation satellite in a sun-synchronous orbit of the kind we use today was Landsat 1 (launched in 1972, then called ERTS-1; 917 km, 99.1°). Since then it has been the standard orbit for imaging, because the same local time gives comparable lighting from one pass to the next; a mid-morning descending node (about 10:30) balances haze and shadows for optical imaging, a dawn-dusk

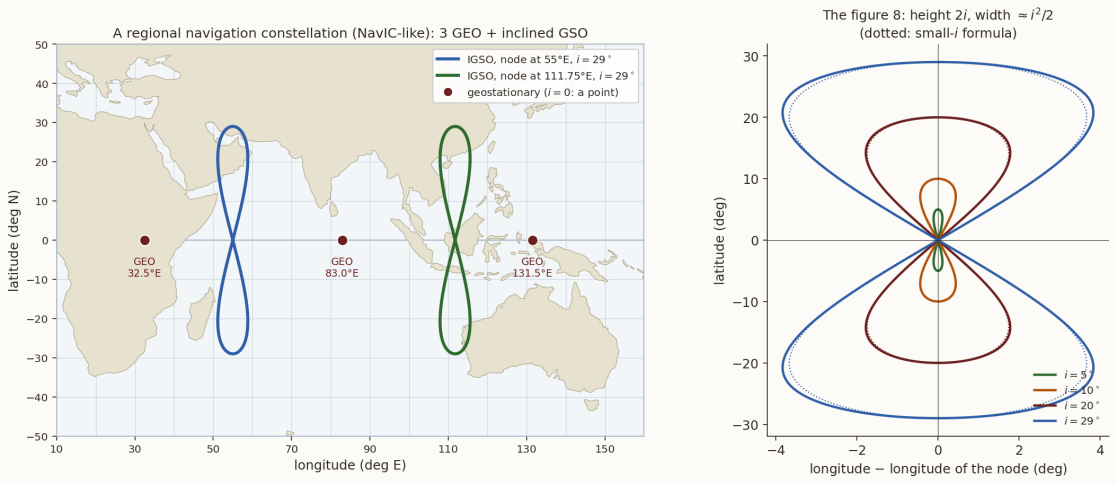


Figure 8: **Left:** a NavIC-like constellation: three geostationary satellites (dots) and two of the inclined geosynchronous figures of eight ($i = 29°$, nodes at 55°E and 111.75°E). **Right:** the shape of the figure 8 for several inclinations, exact (solid) and small-angle formula Equation 9 (dotted). *Script: fig-gt-tracks.py.*

3.5 Highly elliptical orbits: Molniya and Tundra

For $e > 0$ the satellite is not uniformly fast: by Kepler's second law it lingers near apogee. The **Molniya** orbit (T = half a sidereal day, $e = 0.74$, $a = 26\,562$ km, perigee altitude 528 km, apogee altitude 39 840 km) exploits this. Its inclination is $i = 63.4°$, the *critical inclination* $\arccos(1/\sqrt{5})$ at which $\dot{\omega} = 0$ in Equation 10, so the apogee stays over the northern hemisphere instead of rotating away. The satellite is above 55°N for 6.1 h of each 12 h orbit, and the speed at perigee is 6.7 times that at apogee (Figure 9, dots are one hour apart). Two such orbits per day put apogee alternately over Russia (90°E in the figure) and over North America. Geostationary satellites at high latitude are near the horizon; a Molniya satellite is almost overhead. The **Tundra** orbit (T = one sidereal day, $e \approx 0.27$, same inclination) gives a figure 8 with a small northern loop that dwells over one region for many hours.

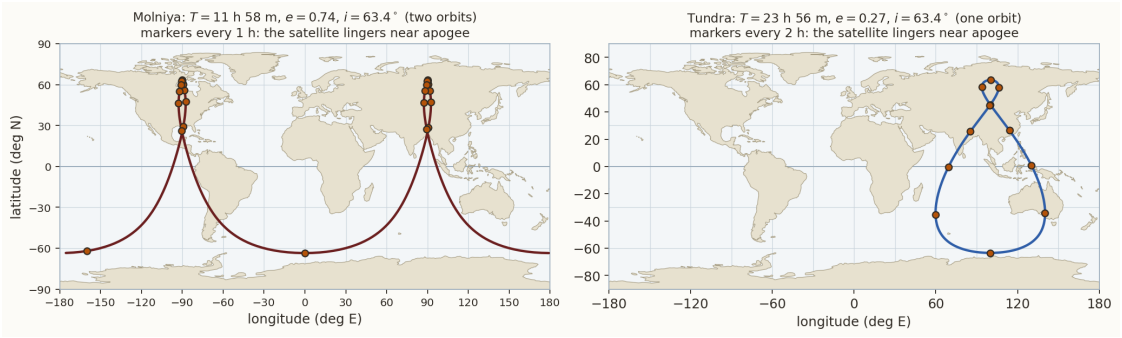


Figure 9: Molniya (two orbits) and Tundra (one orbit) ground tracks, with markers at fixed time intervals. The markers crowd together near apogee, where the satellite is slow. *Script: fig-gt-tracks.py.*

3.6 Repeat ground tracks

A ground track **repeats** if, after k revolutions, the Earth has turned through a whole number D of complete turns *relative to the orbit plane*. Between two successive ascending nodes, the Earth turns relative to the plane through $(\omega_\oplus - \dot\Omega)\,T_\Omega$, where $T_\Omega = 2\pi/\dot u$ is the nodal period ($\dot u = \dot M + \dot\omega$ is the rate of the argument of latitude). Repeat after k orbits and D turns means

$$k\,(\omega_\oplus - \dot\Omega)\,T_\Omega = 2\pi D \quad\Longleftrightarrow\quad \boxed{\frac{\dot M + \dot\omega}{\omega_\oplus - \dot\Omega} = \frac{k}{D}} \tag{12}$$

with k, D integers without common factor. With J_2 neglected this reads $n\,T_{\text{sid}} = 2\pi k/D$, i.e. $T = (D/k)\,T_{\text{sid}}$. The left-hand side of Equation 12 depends on a (and weakly on e and i), so for given k, D, i, e it is solved for a by bisection. The ground track repeats after D *nodal days*, $2\pi/(\omega_\oplus - \dot\Omega)$ each. For a sun-synchronous orbit $\omega_\oplus - \dot\Omega$ is exactly 2π per mean solar day, so the nodal day *is* the solar day (time note) and the cycle is D solar days; for a geosynchronous or GPS orbit $\dot\Omega$ is tiny and the nodal day is very nearly the sidereal day.

After the cycle the k ascending nodes are equally spaced around the equator, $360°/k$ apart: the tracks fill in a grid with spacing $2\pi R_\oplus/k$ at the equator. This is the design equation of an Earth-observation mission: choose k so that the spacing is a little less than the swath of the instrument.

We tested the solver against orbits whose parameters are published (Table 1): the agreement is a check on the whole chain of Equation 10 and Equation 12.

History. The first Molniya satellite was launched in April 1965. The reasoning was geometric: geostationary satellites are near the horizon at the high latitudes of the Soviet Union, and the highly elliptical orbit at the critical inclination is the cheapest way to stay high in the sky for most of the day.

Table 1: Repeat-orbit solver against published orbits. The solver returns the *mean* semi-major axis, so differences of a few kilometres (Landsat: 5 km) against osculating values are expected (Section 2.2). Cycle lengths: Jason-3, 127 orbits in 10 nodal days is 9.9156 solar days because its node regresses at -2.08°/day.

Mission	k/D	Computed $a - R_\oplus$	Published
Sentinel-1 (SSO)	175/12	692.8 km	693 km
Sentinel-2 (SSO)	143/10	786.1 km	786 km
Resourcesat / IRS-1C (SSO)	341/24	817.0 km	817 km
Landsat 8, 9 (SSO)	233/16	699.6 km	705 km (osculating, at the equator)
Jason-3 (66.04°)	127/10 nodal days	1336.3 km, cycle 9.9156 solar d	1336 km, 9.9156 d
GPS (55°)	2/1	20 182.3 km, $a = 26\,560.4$ km	$a = 26\,560$ km
Galileo (56°)	17/10	$a = 29\,600.3$ km	$a = 29\,600$ km
GLONASS (64.8°)	17/8	$a = 25\,507.6$ km	$a = 25\,508$ km

The left panels of Figure 10 show the Landsat-like orbit. It flies 14.56 orbits per day, the ascending nodes advance 24.72° westward per orbit, and after 16 days the 233 tracks are spaced $1.545^\circ = 172.0$ km apart at the equator. The instrument swath of 185 km then gives a sidelap of $(185 - 172)/185 = 7\%$ at the equator, which grows toward the poles. Neighbouring tracks are flown **seven days apart** (modulo the 16-day cycle), which is why a place is not re-imaged for a week after its neighbour. Panel (d) shows the case of GPS: two orbits per sidereal day, so that each day retraces the same path over the ground.

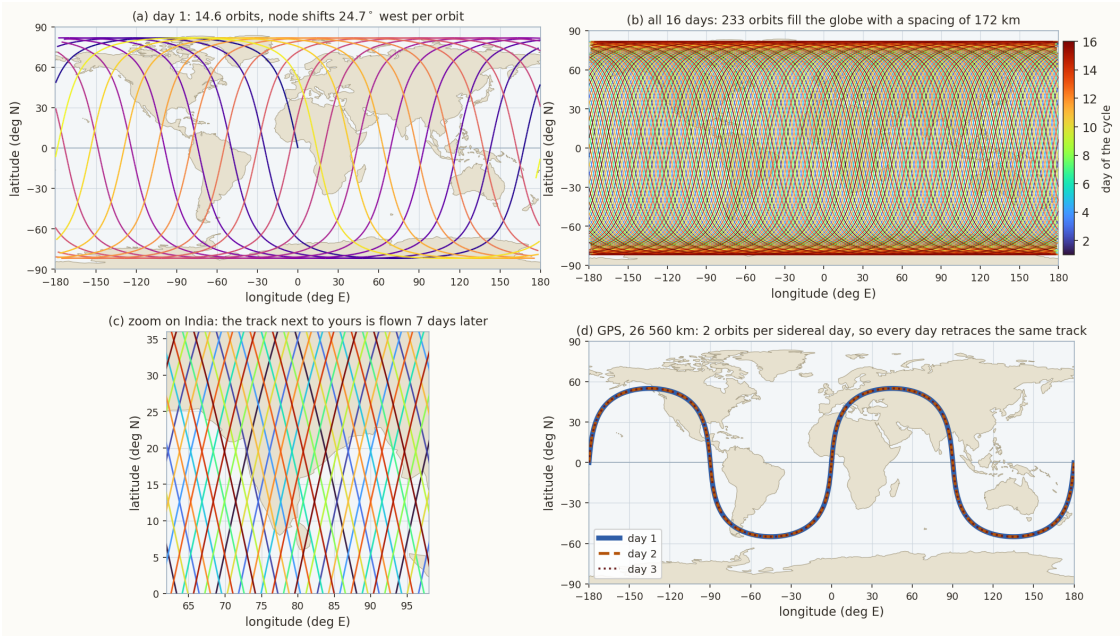


Figure 10: (**a**) One day of a Landsat-like sun-synchronous orbit (233 orbits in 16 days). (**b**) The whole 16-day cycle, coloured by day. (**c**) Zoom on India: adjacent tracks belong to days that differ by 7. (**d**) GPS, three days: all three days coincide. *Script: fig-gt-repeat.py.*

4 An interactive ground-track simulator

The figure below implements the six steps of Section 2 in JavaScript, in the browser. Pick a preset, or change the elements with the sliders (or type exact values into the boxes), and the ground track is redrawn. Press *play* to fly the satellite along its track with its footprint. Tick *ground station* to see the passes. The panel of read-outs gives the derived quantities of this note: period, nodal drift, shift per orbit, and, if the orbit repeats, the ratio of Equation 12.

Figure 11: **Interactive ground-track simulator.** All elements are *mean* elements. The longitude of the ascending node at $t = 0$ is measured with the Greenwich angle set to zero at $t = 0$ (Equation 3 with $\theta_0 = 0$), and, with the *include* J_2 box ticked, the secular rates of Equation 10 are used. The track is coloured by time (blue to red), or by orbit number. The green dot marks the start and the square the end of the plotted interval. The orange disc is the footprint (Section 5) for the chosen minimum elevation angle. With a ground station, the parts of the track above the minimum elevation are drawn thick, and the dashed circle is the region from which the satellite is visible at the current altitude. *How it works:* gt-engine.js has no dependencies and implements Equation 1 to Equation 4, Equation 10 and Equation 12; gt-app.js draws on a canvas; the coastline is Natural Earth 1:110 m land data (make-land-data.py). The same equations, in Python (_gt.py), produced every static figure of this part, and the two implementations agree to 2×10^{-9} degrees on the seven test orbits we compared (circular, elliptical, retrograde, geostationary, with and without J_2).

History. The altimetry satellite TOPEX/Poseidon (1992) was given a 9.9156-day repeat cycle (127 orbits) so that the tidal constituents, which are aliased by the sampling, appear at distinct periods (the lunar M_2 tide is aliased to about 62 days) and can be separated from the slow sea-level changes that the mission was after. Jason-1, -2 and -3 fly the same

Things to try (each takes a minute and each answers a question of this note).

1. *ISS.* Play the preset for a day. How many orbits does it make? How far west does the track move on each orbit (read-out)? Compare with Equation 7.
2. *Geostationary*, then increase i from 0 to 29° in small steps. Watch the point become a figure 8. Reproduce Equation 9.
3. *A 24-hour orbit.* Set the preset "A 24-hour orbit". The track is a straight line along the equator, running west at 0.9856°/day (time note, figure). Change a to 42 164.17 km to stop it.
4. *Sun-synchronous.* Take the SSO preset, untick J_2: the node no longer moves and the local time of the pass is not constant. Tick it again and change i by 0.5°: the read-out shows the drift $\dot{\Omega}$ leaving 0.9856°/day.
5. *Repeat orbit.* Press *solve for a* with $k = 233$, $D = 16$ (sun-synchronous ticked). Play the 16-day cycle. Then type $k = 15$, $D = 1$ with the box unticked: an orbit with 15 revolutions per (nodal) day.
6. *Molniya.* Set $\omega = 270°$, then $\omega = 200°$ and compare. Keep J_2 ticked and change i from 63.4° to 55° or 70°: the apogee drifts ($\dot{\omega} \neq 0$) everywhere except at the critical inclination 63.4°. (With J_2 unticked the apogee never moves, at any inclination.)
7. *A station.* Tick *ground station* at (13.0°N, 77.6°E) and read the passes of the SSO preset. Then move the station to Svalbard (78.2°N, 15.4°E): how many passes per day now?

5 What ground tracks are used for

Table 2: Applications of ground tracks.

Application	Orbit	Ground-track property used	Example
Optical / radar Earth observation	sun-synchronous, repeat orbit	same local time every pass; global coverage; track spacing < swath	Landsat, Sentinel, Resourcesat
Altimetry	non-sun-synchronous, exact repeat	repeat cycle with aliasing of tides	TOPEX/Poseidon, Jason
Continuous regional service	geostationary	fixed sub-satellite point	INSAT, GOES, Meteosat
Regional navigation	GEO + inclined geosynchronous	figure 8 over the region	NavIC, QZSS, BeiDou

Application	Orbit	Ground-track property used	Example
High-latitude communication	Molniya, Tundra	slow near apogee over the north	Molniya, Sirius XM
Global navigation	MEO, repeat every few days	track repeats every sidereal day (GPS), every 8–10 days (GLONASS, Galileo)	GPS, Galileo
Weather, global	polar / SSO	all latitudes, twice a day	Metop, NOAA
Crew and cargo	ISS, 51.6°	visible from most populated latitudes; launch from mid-latitude sites	ISS

5.1 Coverage: footprints and swaths

A satellite at altitude h sees, above a minimum elevation angle ε (in this section ε is an elevation), a circle on the ground of angular radius ψ (the *Earth central angle*) around the sub-satellite point (Figure 12, left). From the triangle Earth centre–station–satellite,

$$\sin\eta = \frac{R_\oplus}{R_\oplus + h}\cos\varepsilon, \qquad \psi = 90^\circ - \varepsilon - \eta, \tag{13}$$

where η is the nadir angle at which the station is seen from the satellite. At 800 km with $\varepsilon = 10^\circ$ the coverage radius is $\psi = 18.95^\circ$ (2109 km); at $\varepsilon = 0$ it is 27.3°. For a geostationary satellite $\psi = 81.3^\circ$ at $\varepsilon = 0$: it sees $\frac{1}{2}(1 - \cos 81.3^\circ) = 42.4\%$ of the whole surface. Three of them, 120° apart, cover all longitudes up to a latitude of about $\pm 72^\circ$ at $\varepsilon = 0$, and about $\pm 62^\circ$ if 5° of elevation is required. For an *imaging* satellite the footprint is the instrument swath, about 100 to 200 km, and the ground track is its centre line: the design condition is the spacing of the repeat grid (Equation 12) against the swath.

5.2 Ground stations: how often, and for how long

The same equation gives the contact with a ground station: the satellite is visible when its sub-satellite point is within ψ of the station (Figure 12, right;

the dashed circle in the simulator). We counted passes above 5° elevation over ten days by direct computation (Table 3).

Table 3: Ground-station passes over ten days (elevation > 5°).

Station	SSO 800 km: passes/day	contact min/day	min per pass	ISS: passes/day	contact min/day
Equator	3.9	37	9.5	3.4	23
Thiruvananthapuram (8.5°N)	3.7	37	10.0	3.6	23
Bengaluru (13.0°N)	3.8	37	9.8	3.6	24
Delhi (28.6°N)	4.3	42	9.8	4.6	30
Svalbard (78.2°N)	14.3	156	10.9	0	0

A polar-type orbit passes near the poles on every one of its 14.3 orbits per day, so a station at 78°N is in contact on *every* orbit: this is why polar-orbit missions use high-latitude stations for data download. A station at low latitude sees such a satellite only three to four times a day, each pass lasting about ten minutes. The ISS never rises above 5° from Svalbard. Two practical numbers follow: a low-latitude station can download a few tens of minutes of data per day from a polar LEO satellite (which is why on-board storage and a high-latitude station matter), and a low-inclination LEO satellite is best served by stations near its maximum latitude.

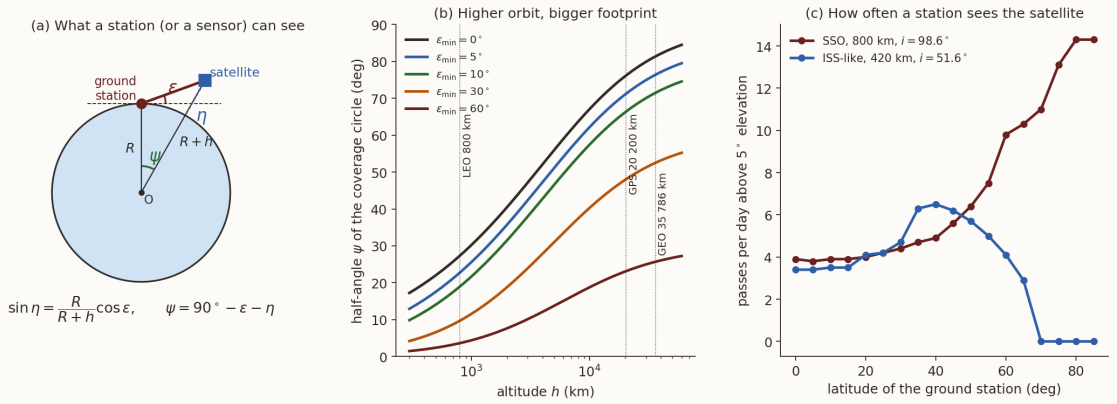


Figure 12: (**a**) The geometry of Equation 13, with the altitude exaggerated. (**b**) The half-angle ψ of the coverage circle against altitude for several minimum elevations. (**c**) Passes per day above 5° elevation against the latitude of the ground station, for a sun-synchronous orbit at 800 km and for the ISS. *Script: fig-gt-apps.py.*

5.3 The launch site sets the lowest inclination

A launch vehicle can put a satellite directly into an orbit of inclination i only if $i \geq \varphi_L$, the latitude of the launch site, because the orbit plane contains the Earth's centre and the launch point, so it reaches at least latitude φ_L. The launch azimuth A_z (from north, inertial) follows from the spherical triangle of Figure 3 applied to the launch point:

$$\cos i = \cos \varphi_L \, \sin A_z . \tag{14}$$

A due-east launch ($A_z = 90°$) gives $i = \varphi_L$ and also takes full advantage of the Earth's rotation, which contributes $\omega_\oplus R_\oplus \cos \varphi_L$ to the speed: 463 m/s at Kourou (5.2°N), 452 m/s at Sriharikota (13.7°N), 409 m/s at Cape Canaveral (28.5°N), 324 m/s at Baikonur (45.9°N). For a sun-synchronous orbit with $i = 98.7°$ from Sriharikota, Equation 14 gives $A_z \approx -9°$ or 189°: the vehicle flies almost due south, against the rotation of the Earth, and the ground track through the launch site heads 193°, south-south-west, after the rotation of the Earth is taken out (Figure 13, right). This track passes close to the west coast of Sri Lanka, and avoiding an overflight of Sri Lanka is the reason usually given for the *dog-leg* manoeuvre that India's PSLV flies on its sun-synchronous launches (the ascent path, unlike the final orbit's track, is bent around the island; our figure shows only the orbit's ground track through the pad).

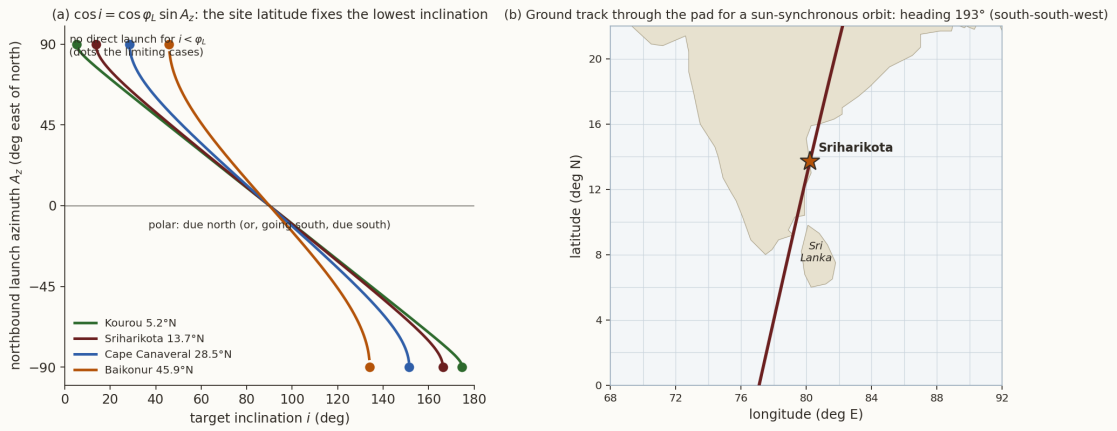


Figure 13: (**a**) Launch azimuth against target inclination for four launch sites; each curve starts at the point ($i = \varphi_L$, due east) and ends at ($i = 180^\circ - \varphi_L$, due west). The southbound azimuth is $180^\circ - A_z$. (**b**) The ground track through Sriharikota for an 820 km sun-synchronous orbit. *Script: fig-gt-apps.py.*

Summary and outlook

In one paragraph. The ground track is the inertial orbit seen from the rotating Earth: elements → Kepler → perifocal → inertial → rotate by $\theta = \theta_0 + \omega_\oplus t$ → (φ, λ). For circular orbits $\sin\varphi = \sin i \sin u$ and $\tan(\alpha - \Omega) = \cos i \tan u$: the highest latitude is i, the track moves $360^\circ\, T/T_{\text{sid}}$ west per orbit, and near the geosynchronous radius a small inclination gives a figure 8 of width $i^2/2$. J_2 adds $\dot\Omega$, $\dot\omega$, $\dot M$ (use mean elements!) and makes sun-synchronous orbits possible; the repeat condition is $(\dot M + \dot\omega)/(\omega_\oplus - \dot\Omega) = k/D$. Every application in Table 2 is a statement about one of these properties.

The numbers that follow from formulas in this note are checked by ground-tracks-verify.py.

What we left out. (i) *Precession, nutation and polar motion.* The transformation from ECI to ECEF is here a single rotation. The full IAU 2006/2000A transformation adds precession, nutation and polar motion, which move points on the ground by up to a few hundred metres. (ii) *Drag and other perturbations*, which change a and hence the period and the repeat pattern: repeat-orbit missions carry propellant to hold the ground track within a tolerance band (typically ± 1 km of the reference track for the Landsat series; check the mission documentation).

Further reading. H. D. Curtis, *Orbital Mechanics for Engineering Students* (ground tracks); D. A. Vallado, *Fundamentals of Astrodynamics and Applications* (ground tracks and repeat orbits, coverage).

Exercises

1. **Heading at the node.** Evaluate Equation 8 for the ISS ($T = 92.97$ min, $i = 51.64°$). What is the heading for an SSO at 800 km? Why is it larger than 90°? (Answers: 54.7°; 102.5°, because the orbit is retrograde and the Earth's rotation adds to the westward motion.)
2. **SSO design.** Find the inclination of a circular sun-synchronous orbit at 700 km from $\dot{\Omega} = 0.9856°$/day. (Answer: 98.19°.)
3. **Track spacing.** For 233 orbits in 16 days, find the equatorial spacing of tracks in degrees and kilometres, and the sidelap of a 185 km swath. (Answers: 1.545°, 172 km, 7%.)
4. **Repeat orbit, no J_2.** Find the altitude of a circular orbit with 15 revolutions in one stellar day, neglecting J_2. (Answer: about 554 km.)
5. **Figure 8.** For a geosynchronous orbit with $i = 29°$ find the width and height of the figure 8 from Equation 9, and compare with the exact value $\pm 3.84°$ of the longitude excursion.
6. **Footprint.** At what altitude does a satellite see an Earth central angle of $\psi = 30°$ at $\varepsilon = 10°$? Use Equation 13. (Answer: $\eta = 50°$, $R_\oplus + h = R_\oplus \cos 10° / \sin 50°$, $h \approx 1820$ km.)
7. **Launch.** Which is the lowest inclination reachable directly from Kourou? From Baikonur? What is the northbound launch azimuth from Sriharikota into a 98.6° orbit? (Answers: 5.2°; 45.9°; $-8.9°$, i.e. 351.1°.)
8. **Simulator.** Using Section 4, find the altitude and inclination of a sun-synchronous orbit that repeats after $D = 16$ solar days with $k = 229$ orbits, and read off the spacing of the tracks ($360°/229 = 1.572°$). Why would $k = 232$, $D = 16$ not be a good choice? (Hint: $\gcd(232, 16) = 8$.) (Answers: $a = 7160.1$ km, altitude 781.9 km, $i = 98.53°$, spacing 175 km; $232/16 = 29/2$ repeats after 2 days, not 16.)